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THE STUDY OF THE PHYSICS OF COMETARY NUCLEI

Grant NSG 7082

Semiannual Progress Report No. 3

1 September 1975 to 29 February 1976

Principal Investigators

Dr. Fred L. Whipple
Dr. Brian G. Marsden
Dr. Zdenek Sekanina

Prepared for

National Aeronautics and Space Administration
Washington, D.C. 20546

March 1976

Smithsonian Institution
Astrophysical Observatory
Cambridge, Massachusetts 02138

The Smithsonian Astrophysical Observatory
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THE STUDY OF THE PHYSICS OF COMETARY NUCLEI

Semiannual Progress Report No. 3

Individual reports on the progress of research by F. L. Whipple, B. G. Marsden, and Z. Sekanina follow.

PHYSICS OF COMETARY NUCLEI

Whipple's paper "A Speculation about Comets and the Earth," presented at the Twentieth Liège International Astrophysical Colloquium at Liège, Belgium, on June 19, 1975, has appeared as Center for Astrophysics Preprint No. 370. It was presented at the PPPI meeting at Flagstaff and will eventually be published in the volume of the Liège Colloquium. Work on this intriguing subject is postponed because of other research and writing commitments, some mentioned below.

With Walter F. Huebner, Whipple has completed a chapter entitled "The Physics of Comets" for Reviews of Astronomy and Astrophysics, 1976, which has appeared as Center for Astrophysics Preprint No. 413 (Attachment 1).

His earlier reported paper on "Criteria for the Identity of Comet Orbits" with M. Lecar is being revised and will first appear as a Center for Astrophysics Preprint.

Whipple's research on split comets continues and will probably soon be presented in a three-part series of papers featuring:

A. A report on the study of the phenomena of split comets, which shows that gravitationally double comets are an unlikely cause for those not tidally disrupted and that jet-action "spin-up" appears to be the most likely of the intrinsic splitting mechanisms.

B. A report on a search for real pairs of comets among the orbits of parabolic and very long-period comets with inclinations in the range 70-110°.

C. An analysis of E. J. Öpik's comprehensive study "Comet Families and Trans-neptunian Planets" (Irish Astr. J., 10, 35-92, 1971) has been completed and a copy is attached: "The Reality of Comet Groups and Pairs" (Attachment 2). It will be published

as a Center for Astrophysics Preprint as soon as a journal for publication has been selected. The results of this study were presented briefly at the PPPI meeting at Flagstaff, Arizona, in March. By statistical methods Öpik proves to his satisfaction that most very-long period comets occur in a number of orbitally associated groups that must have involved either multiple comet splitting or large original cohesive groups in the Öpik-Oort cloud. Whipple finds by the Monte-Carlo method of statistics and by probability theory that the groups containing more than two comets are statistically unreal. Possibly some of the pairs are real, a matter that will be investigated under B above.

Whipple has written a long chapter entitled "Comets" for a Wiley book on Cosmic Dust (not a part of the project). In the course of this effort he has found a new application of nongravitational solar-radial forces as a measure of comet nucleus dimensions and activity. An extension of this work will be pursued during the next months. Very old comet nuclei behave in the expected fashion for dirty-ice models but some long-period comets (e.g., C/Bennett, 1970 II) show an unexpectedly large solar repulsive acceleration. An explanation for this phenomenon should provide valuable new information about the nature of comet nuclei.

F. L. Whipple

ORBITAL CALCULATIONS

Marsden and Sekanina have continued to improve the orbits of long-period comets. It is expected that much of this work and a complete revision of the tabulation of "original" reciprocal semimajor axes will be prepared for publication during the next few months.

Marsden has also improved the orbits of the nine new short-period comets discovered since late 1972 and has continued his work on nongravitational effects. He attempted to link the two apparitions of P/Brorsen-Metcalf (1847 and 1919) but ran into problems similar to those previously experienced with P/Westphal. Both comets have revolution periods of 60-70 years, and although P/Brorsen-Metcalf did not show the curious physical behavior exhibited by P/Westphal at its 1913 return, it seems probable that the problems have some physical, rather than mathematical, cause.

B. G. Marsden

EMISSION OF LARGE PARTICLES FROM COMETARY NUCLEI

Sekanina has completed his study of anomalous tails of comets observed in the past near the "edge-on" projection, i.e., near the time of the earth's passage through the comet orbit plane. This paper has appeared as Center for Astrophysics Preprint No. 445. The abstract of the study is attached (Attachment 3).

Work is currently in progress on Part II of the study, which elaborates on prospects for observing anomalous tails in the future returns of the short-period comets. When this research is completed – within a few months from now – the whole study will be submitted for publication, probably – because of its length – in a journal's supplementary series.

OTHER ACTIVITY

Sekanina's paper on the probability theory of encounter with interstellar comets, completed in the previous period, has now been published [Icarus, vol. 27 (1976), pp. 123-133].

Significant progress has been made in the study of the band structures in dust tails of comets. Additional photographs, showing the structures in Comet Mrkos 1957 V, were acquired courtesy of Dr. A. R. Klemola, Lick Observatory. Work has advanced on the photographs obtained by A. McClure (cf. Semiannual Progress Report No. 2). Most importantly, a digital processing routine has been developed in the Los Alamos Scientific Laboratory, Los Alamos, New Mexico, aimed at enhancing the details of the band structures. The work has been done at the request of Dr. J. A. Farrell, who furnished some of the photographs of Comet 1957 V and who is actively collaborating with Sekanina on the problem. The computer technique, developed at absolutely no cost to the present NASA grant, will shortly be applied – also free of charge – to all gathered photographs of comets 1957 V and 1910 I.

Z. Sekanina

ATTACHMENTS

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No. 413

PHYSICAL PROCESSES IN COMETS

F. L. Whipple and W. F. Huebner

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October 1975

PHYSICAL PROCESSES IN COMETS

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1. INTRODUCTION

Comets are intrinsically of interest as unique and often spectacular members of near space. They are even more important as representing the most primitive material of the solar system, eventually available for direct study by space probes. They are now providing important information as to the chemistry, physics and processes involved in the formation of the solar system and probably represent boundary conditions for general theories of star formation. Comets provide the major contribution to the interplanetary complex, possibly including some small asteroids. The shape of the cometary Lα isophotes can be used to measure the solar Lα flux. Comets may well have spread a coating of volatiles on the Earth, essential to the origin of life. Comets may yield clues about interstellar molecules. They are useful as space probes, especially if they develop ion tails at large heliocentric distances or if their orbits place them well out of the ecliptic plane or lead through the solar corona. Possibly comets play a role in galactic chemistry (Tinsley and Cameron, 1974 and Whipple, 1975). Thus the study of comets is desirable for a variety of reasons.

[1]This research was partially supported by Grant No. NSG 7082 from the National Aeronautics and Space Administration

[2]Work performed under the auspices of the U.S. Energy Research and Development Administration.

We assume that the nucleus of a comet consists of ices, clathrates (molecules or atoms bonded in ice such as H_2O particularly) and grains of rocky (meteoroidal) material intimately mixed with the ices and clathrates. Under the activation of solar radiation -- reduced by reflection, attenuation near the surface, heat conduction into the surface, and reradiation -- the rate of sublimation determines the loss of gases directly and the loss of icy-rocky grains carried away by the gases. Rotation of the nucleus is a perturbation on the process, exposing most of the nucleus periodically to sunlight and producing a mean vector of ejection that deviates from the solar direction. The net jet component of this force on the nucleus produces the observed non-gravitational motions observed in practically all comets where orbital data are sufficiently accurate.

The activity of the nucleus is highly dependent upon the intensity of solar radiation and therefore upon solar distance, being extremely sensitive to the vapor pressure--i.e. heat of vaporization--of the ices present, as well as to their physical and chemical association with the refractory solids. These circumstances are weakly inferred from the observations. Note that the material must be extremely weak structurally and highly inhomogeneous for the weak forces of sublimation to destroy the surface. Even at great solar distances, r>4 AU, most observable comets appear fuzzy, the unresolvable nucleus being surrounded by a coma.

For comets with large perihelion distance, q, the coma grows at the time of cometary outbursts. The spectra are continuous, implying the ejection of dust, which probably consists mostly of icy grains.

Thus the true nucleus of a comet is rarely, if ever, observed. The observable phenomena are purely transient, being activated by solar radiation and the solar wind (see Fig. 1). The gases and meteoroids are lost forever from the comet. At $r \gtrsim 2.5$ AU, ices more volatile than H_2O ice are active, while below $r \sim 2.0$ AU H_2O ice appears to have a major influence on the cometary sublimation and ejection processes. For most comets fluorescent bands of molecules appear: first the (0-0) band of CN at $r \sim 3$-2.5 AU, then C_3 and NH_2 bands at about 2 AU, C_2 (Swan bands) at ~ 1.8 AU, the Na D-lines at ~ 0.7 AU and other atomic lines closer to the Sun (see Table I). For a few comets a continuous spectrum arising from direct reflection by particles in the coma predominates even at small solar distances, but the major radiation, optical and UV, is usually in the molecular bands of radicals by fluorescence. In the far ultraviolet, the Lα line of hydrogen overwhelms the spectrum, while in the infrared, scattering and radiation from solid particles prevails. Radiation pumping may play an important role for several radio transitions. Absorption of solar radiation occurs in discrete ground or metastable levels of the radicals and atoms, the observed

spectra depending upon the specific configurations of higher molecular and atomic levels. Band structures are distorted by irregularities in the solar spectrum that affect the transitions in the critical wavelengths absorbed, depending on the Doppler effects of radial velocity with respect to the Sun and of gas motions within the coma.

Thus within the central coma, the physical processes involve the radial forces from the nucleus and from the Sun (radiation pressure and the solar wind) affecting the motions of the gases and grains, gas-phase chemistry very near the nucleus of large comets near the Sun, sublimation from the grains, dissociation of mother molecules into radicals and radicals into atoms, charge exchange with solar-wind protons, excitation by sunlight resulting in fluorescent radiation, ionization by solar radiation and by collision, and finally, the scattering of sunlight by all types of particles. Light pressure on the smaller solid particles propels them into dust tails visible in most bright comets, lagging many degrees behind the antisolar direction because of the conservation of orbital angular velocity.

Both solar radiation and the solar wind ionize radicals and atoms in the coma. The ions are then susceptible to forces from the solar wind via chaotic magnetic fields generated in the mixing zone behind the bow wave as the solar wind interacts with the coma gases. At a solar wind expansion velocity of some 350 km/sec

the solar wind carries adequate momentum to accelerate the ions away from the comet to velocities from ∿10 to ∿200 km/sec, producing the great ion tails so conspicuous in bright comets. The main axes of ion tails lie nearly in the orbital plane but deviate from the antisolar direction by the "aberration" angle between the solar wind and the transverse cometary velocities. The plasma physics involves a number of processes, not all quantitatively understood as yet.

In the following sections of this paper we discuss most of the tractable processes mentioned above, relating them to cometary observations. Many other processes relevant to comets, such as the tidal splitting of sun-grazing comets, internal processes of radioactivity, chemical changes, heat transfer and gas transfer, and external interaction with high-energy particles such as cosmic rays will not be discussed. We will discuss cometary origin briefly and the relation of comets to the Earth, to the interplanetary complex, and to the interstellar medium. Finally, we will mention future desirable observations, especially by space missions to actual comets.

Although there is no viable taxonomy of comets we shall occasionally use the adjective "new" to designate comets with extremely large semi-major axes (believed to be making their first journey to the inner solar system from the Öpik-Oort

cloud) and "old", relating to comets of short period that have experienced many inner-solar-system passages. This usage was initiated by Oort and Schmidt (1951).

2. NUCLEUS

Aside from the directly observable phenomena of comet comae, tails, etc., we learn about the nuclei by studying their contributions to the interplanetary complex. When the orbit of a comet passes fairly close to the Earth's orbit we usually observe associated meteors and, in several cases, spectacular streams. The meteoroids released from comets further maintain the cloud of particles that produce sporadic meteors, the Zodiacal Light and the Gegenschein, the latter two observed by scattered sunlight. Some 10-30 tons per second of cometary solids are required to maintain a quasi-stable equilibrium (Whipple, 1967). The material of the meteoroids has roughly solar composition ratios for Na, Mg, Ca, and Fe (Millman, 1972) and low densities, typically 0.8 gm/cm^3, but ranging far below this value to somewhat higher values (Verniani, 1975). They are very friable, weak structures in the main; some, if recoverable, could be crushed between the fingers. In the interplanetary complex the meteoroids are destroyed by collisions, most of the mass ending up as ions, carried to interstellar space by the solar wind.

Thus, the solids in comets, studied mostly as transient meteors, seem to be physically consistent with particles that were formed or trapped in an icy mix, not constituting a coherent mass of rocky solids penetrated by gas or liquids that froze to ices. Considerable additional information about the solids comes from studies of the dust coma and tails and the antitails (see Sects. 4 and 7), supporting the theory by Huebner and Weigert (1966) that the nucleus ejects icy-grains into the coma, and consistent with observed porous meteoroids, observed after they are "de-iced" and degassed.

The persistence of some sun-grazing comets with perihelia within the lower corona or upper chromosphere prove a certain degree of compressive strength in the nuclei, estimated at 10^4-10^6 dyne cm^{-2} by Öpik (1966a). Very little tensile strength, however, is required to resist the tidal disruptive forces. On the other hand, the parent body for the Kreutz sun-grazing family of 7 or 8 members must surely have been split by a close solar passage, and various members were observed to lose short-lived pieces near perihelion. About a dozen comets have split at varying solar distances not related to perihelion (the list includes some "new" comets) at first apparitions from the Öpik-Oort cloud (Öpik, 1932; Oort, 1950). The evidence (Whipple unpublished) does not favor the occurrence of gravitationally bound double or multiple nuclei. Presumably the non-tidal

break-ups arise from too rapid rotation, i.e. spin-up induced by non-symmetrical jet forces from the sublimation of ices.

Variations in intrinsic brightness, commonly called <u>bursts</u> when exceeding a magnitude or two, are frequent among all classes of comets. The most notable case is P/Schwassmann-Wachmann with period P=15 yr. and q=5.4 AU, which occasionally brightens by several magnitudes. The most violent example, P/Tuttle-Giacobini-Kresák (P=5.6 yr, q=1.2 AU) increased by 9 magnitudes twice in 1973 (see Kresák, 1974). Here the cause may be a break-up or crushing of a considerable volume of friable material exposing volatiles to sunlight induced by large-scale inhomogeneities in the structure of the nucleus ("hot spots", see Sekanina, 1972) and irregular wasting of the material, leading to cave-ins. The tendency for large bursts of small comets to occur in pairs separated by months, pointed out by Kresák, suggests changes in the moments of inertia of the nucleus, resulting in a realignment of the rotation axis, with subsequent internal stress changes that produce break-up. Possibly ordinary asymmetrical wasting by sublimation can initiate the process. For large comets, however, inhomogeneities must be the major cause, allowing for the possibility of very rare encounters with small bodies (Harwit, 1967) and exothermic reactions among atoms or radicals (see Donn and Urey, 1956).

The nature of the volatiles producing comet activity and the physical association with meteoroidal material must vary.

strikingly from comet to comet, judging by the highly individual laws of intrinsic brightness as a function of solar distance. This much discussed subject will be treated only superficially here because of lack of space. The absolute brightness (reduced to 1 AU from the Earth) of a comet can be approximated by a constant/r^n. The exponent n averages around 3.3 for all comets but frequently ranges from <2 to >6, with exceptional cases of negative values (fading on the way to perihelion) to very high positive values. Nor is n constant for a given comet, sometimes changing abruptly, often near perihelion. The evidence suggests that for old periodic comets, sublimation activity is often confined to small areas of the nucleus, varying with time, and that volatiles of low vapor pressure such as H_2O ice are major constituents. For new comets at their first apparition (C/Kohoutek, 1973 XII, for example), the outer surfaces appear to contain compounds much more volatile than H_2O ice, producing activity at great solar distances, nearly to Jupiter's distance.

One might expect the dimensions of cometary nuclei to be deducible from their reflected sunlight when inactive at great solar distances. For this only three assumptions are needed: albedo, A, corrections for phase angle, and inactivity; these, coupled with observed magnitudes at known distances, should be adequate. Roemer (1966) presents nuclear radii for 29 comets,

but questions the assumption of inactivity or complete lack of coma even for star-like images in a large reflector. The measures give radii in the range 0.1 km (A=0.7) for the faintest to 40 km (A=0.02) for the largest. The mean radius (A=0.7) is 1.0 km for 19 periodic comets and 2.4 km for 10 nearly parabolic, varying as $(0.7/A)^{1/2}$ for other assumed values of the albedo. These values can be taken as upper limits in view of Sekanina's (1974a) strong arguments that distant comets may rarely be devoid of comae.

III. COMA: GAS PRODUCTION RATES

Direct studies of the nucleus must await observations from space missions to comets. Until then observations of the coma yield the richest information from which deductions can be made about the structure and composition of the nucleus.

Solar radiation incident on the nucleus is in part reflected by the surface, in part absorbed by the thermal surface layer and some possibly transmitted to a small depth. Most of the absorbed radiation is either reemitted in the infrared or used to sublimate frozen gases from the nucleus. For "old" comets possessing an appreciable refractory crust, a significant fraction may be conducted deeper into the nucleus (see Sekanina, 1969). The partitioning of energy into the various processes depends strongly on the material properties of the nucleus. Its fluffy

structure is thought to be more akin to snow which has quite different properties than clear ice. The amount of reflected solar radiation depends on the albedo,* i.e., on the result of multiple scattering of light by the grainy structure of the surface ice. The albedo therefore depends also on the angle of incidence of the light. Investigations of the albedo of antarctic snow and ice carried out by Barkstrom (1972) may also be quite relevant to comet nuclei. His values for the albedo range from 0.72 ± 0.03 at $\cos\theta = 0$ to 0.88 ± 0.02 at $\cos\theta = 1$, and vary nearly linearly with $\cos\theta$; θ is the angle between the normal to the surface and the incident radiation. Emission in the infrared depends strongly on temperature, chemical composition, crystal structure, and small-scale geometry of the surface, (plane emissivity, hemispherical emissivity, etc., see, e.g., Penner and Olfe, 1968). Sublimation depends on the material's latent heat and its temperature dependence. Heat flow into the interior of the nucleus depends on the radiative and conductive properties of the substance involving its structural composition and the ratio of meteoritic material to ices. Properties for materials that likely exist in comets have been summarized by Pounder (1965), Huebner (1965, 1970), Delsemme and Miller (1970), and Lebofsky (1975). Thermal conductivity of ice has been investigated by Klinger and Neumaier (1969) and Klinger (1973).

*Not the Bond albedo which includes the effects of a spherical surface.

The steady state energy equation

$$\int\left(f_\nu/r_h^2\right)e^{-\tau_\nu}[1 - A_\nu(\theta)]\cos\theta\, d\nu = \varepsilon\sigma[T(\theta)]^4 + Z(\theta)L\Big(T(\theta)\Big)/N_o + \kappa\left.\frac{dT(\theta)}{dr}\right|_{R^-}$$

insolation = reradiation + sublimation + conduction (1)

and a Clausius-Clapeyron type equation

$$\log Z(\theta) = \log\left(f_a\ p_o\sqrt{\frac{N_o}{2\pi MkT}}\right) + [\alpha - \beta L\Big(T(\theta)\Big)/T(\theta)], \tag{2}$$

must be solved simultaneously to determine the gas production rate, $Z(\theta)$ [molecules area^{-1} time^{-1} sterad^{-1}], for forming the coma and to predict the steady state temperature, $T(\theta)$, of the coma-nucleus interface (the thermal layer). In the above equations f_a is the accommodation factor, $A_\nu(\theta)$ is the albedo; f_ν/r_h^2 is the solar flux at 1 AU per unit area and per unit frequency interval diluted corresponding to heliocentric distance r_h in AU; ε is the frequency-integrated (primarily infrared) emissivity; κ is the coefficient of heat conduction below the surface R^- of the nucleus; $L(T)$ is the average latent heat of sublimation of the surface material; and M is the average molecular weight. The constants α and β are

$$\alpha = \log\{\exp[L(T_o)/(R_oT_o)]\} \tag{2a}$$

$$\beta = \log [\exp(1/R_o)] = 0.21856 \ , \quad [R_o \text{ in cal deg}^{-1} \text{ mole}^{-1}] \quad . \tag{2b}$$

and p_o and T_o are reference points on the vapor pressure curve (e.g., p_o is atmospheric pressure and T_o the boiling point) and R_o is the gas constant. Equations (1) and (2) are valid for a nonrotating nucleus. In general we also need to consider the spin velocity of the nucleus and the temperature relaxation time which depends on heat capacity. The nearly spherical appearance of the coma indicates that to first order sublimation is isotropic--probably due to the spin of the nucleus. Solutions for some combinations of parameters have been given by Huebner (1965, 1967) and by Delsemme (1966). Solutions for four different combinations of parameters are given in Fig. 2.

Early estimates of the gas production were based on partial densities, such as could be obtained, e.g., from the brightness of the comet in the light of C_2 transitions (Wurm, 1943, 1961). A major revision upward by several powers of ten are required by the icy conglomerate model (Whipple, 1950, 1951) and were made by Biermann and Trefftz (1964). The latter based their predictions on the observation of the forbidden O lines at λ = 5577, 6300, and 6364 Å, visible only in the brightest comets. These results were confirmed by a second prediction based on the solution of the energy balance and Clausius-Clapeyron equations,

given by Huebner (1965). Analysis of the violet band of CN at different distances from the nucleus taking into account collisional effects to excite rotational levels led Malaise (1970) to a third independent method for determining densities--and therefore production rates--giving results in general agreement with the above two earlier procedures. Observation of the Lα emission from the hydrogen coma provides a fourth independent determination of the gas production rates (Keller, 1973; Keller and Lillie, 1974; Keller and Thomas, 1975). The agreement (within a factor of about 10) of all four methods indicates that the total gas production rates are known to within rather limited uncertainties (see Sect. 5 for numerical values), and by virtue of the second method the main features of the physical processes for producing the coma are understood. Thus the icy-conglomerate model finds very strong support from indirect observations. Desorbed gases from grains (once thought to be the dominant mechanism) make only minor contributions (Levin, 1972).

Inhomogeneities of the surface layer (composition of frozen gases with different latent heats, variations of albedo, insulation by local accumulations of refractory grains, exhaustion of the more volatile components, etc.), as noted earlier, give gas production rates that are not uniform in space and time. To date the observed data are insufficiently resolved so that only average rates are considered; (however, some jets of dust and

neutral molecules have been observed--see, e.g., Rahe et al., 1969; Huebner et al., 1974, Huebner, 1975) but clearly, the subsolar point receives a maximum of insolation and the antisolar point a minimum. For a spinning nucleus the time delay to affect vaporization causes an angular displacement of the peak production from the solar direction. The reaction of the escaping gas, the "pin-wheel effect", imparts a "nongravitational force" which was the enlightening thought behind the original formulation of the icy-conglomerate model (Whipple, 1950, 1951). The new mass losses associated with the gas production are of the right magnitude to account for the nongravitational forces (Huebner, 1967). More detailed analysis of orbits for several comets by Marsden (see, e.g., Marsden, 1972; Marsden et al., 1973; and Marsden and Sekanina, 1974) agree with the assumptions of the icy-conglomerate model.

4. COMA: GRAINS

Delsemme and Swings (1952) suggested that molecules, such as CH_4 and CO_2, are stored in the form of clathrates in the nucleus. Clathrates are solid molecular compounds, e.g., H_2O ice, with a lattice structure which encloses and bonds by Van der Waals forces other molecules or atoms (see, e.g., Miller, 1973). Delsemme and Wenger (1970) succeeded in the laboratory (although at higher gas pressures than might be expected in the environment of a comet) to produce such clathrates which they observed as mm and sub-mm sized grains. These icy grains as well as the heavier refractory grains which are embedded in the nucleus will

be dragged into the coma primarily by the more volatile sublimating and escaping gases. An icy-grain coma was first postulated by Huebner and Weigert (1966) to explain the outbursts and ensuing slow decay in brightness of P/Schwassmann-Wachmann (1925II). Icy grains are expected to have a long lifetime at large heliocentric distances. The lifetime, τ_g, of the grains

$$\tau_g = - a_o/\dot{a} = a_o N_o/(MZ) \qquad (3)$$

is determined by their rate of sublimation, Z. Here a_o is the initial (at time of lift-off) "radius" of the grain, M the mean molecular weight, and $\dot{a}$ is the rate of decrease of particle radius. Delsemme and Miller (1970, 1971) have suggested that clathrate ice grains contribute to the coma in a more general way, even at small heliocentric distances. For such comets only large icy grains will have a lifetime long enough to contribute significantly to an ice-halo. At small heliocentric distances the rate of sublimation is increased and the proportionate increase in lift forces can transport heavier particles into the coma. The maximum initial radius, a_m, of a grain that can be lifted off the surface by the escaping gases is

$$a_m \approx \frac{9Mv_r Z}{16\pi \rho_g RN_o G} = 1.4 \times 10^{-13} T^{1/2} Z/R \quad , \quad [\mathrm{cm}] \quad , \qquad (4)$$

(first given by Whipple, 1951, in a slightly different form) where M indicates the mean molecular weight of the gases subliming from the surface of the nucleus, Z is the associated gas production rate, v_r the radial component of the thermal gas velocity, ρ_g the density of the grains, R the radius of the nucleus and G is the gravitational constant. The numerical factor is appropriate for H_2O ice-grains. Adhesion of the grains to the nuclear surface will reduce the value for a_m, while nuclear spin will increase it. Refractory grains will be affected similarly, except their lifetimes play a role only at small heliocentric distances and are of particular importance to sun grazing comets and, the Na production in the tail (Huebner, 1970). Important contributions to these effects were made more recently by Sekanina (1973a); the work carried out by Lamy (1974) on interplanetary grains should also bear directly on the behavior of cometary dust.

Setting $a_o = a_m$ and assuming that the rate of sublimation from the icy grains is the same as from the surface of the nucleus we obtain a maximum value for the grain lifetime from Eq. (3) and (4)

$$\tau_g = \frac{9v_r}{16\pi\ RG} = 4.6 \times 10^9\ T^{1/2}/R \quad , \quad [\text{sec}] \quad . \tag{3a}$$

The proposal of adsorbed gases in an icy-grain halo is an attempt to explain the short lifetime of mother molecules. O'Dell and Osterbrock

(1962), Wurm (1963), Malaise (1966), Miller (1967), Vanysek and Žáček (1967) and Vanýsek (1969a,1969b) predicted--from analysis of comet observations--mean lifes of mother molecules which are much smaller than laboratory values predicted by Potter and Del Duca (1964). Typically clathrates contain 6 to 17 H_2O molecules for each other molecule or radical occluded in a cage (Miller, 1973). Knowledge about molecular abundance ratios in the coma is still too sparse and uncertain to serve as indicator whether all observed radicals could be the result of stripping from clathrates. However, Delsemme and Miller (1971) found it necessary to assume that sublimation from the nucleus is controlled by a more volatile component than water or clathrates in order to increase the gas production rates so that larger grains than indicated by Eq. (4) can be lifted off the surface. This allows for a longer lifetime of the grains and increases their range at which they release their trapped radicals to be in better agreement with observed mean life of the mother substance. A depletion of the more volatile component not bound in clathrates will reduce the apparent lifetime of the mother molecules, but no change in lifetime from before to after perihelion (at the same heliocentric distance) has been reported.

Variation of abundance of radicals with heliocentric distance as obtained from spectroscopic data can be used to determine the latent heat of sublimation of the mother molecule using Eqs. (1) and (2). If the radical is released from a

clathrate its latent heat will be larger (and the change in brightness steeper with changing heliocentric distance) than if it is sublimated directly from the nucleus. Spectroscopic data for this purpose must be collected under identical conditions for a period of time covering as large a range of heliocentric distance as possible.

By discussing lifetimes of grains we have already indicated some destructive mechanisms for the coma. Molecules, radicals, atoms, and ions are also destroyed by dissociation, ionization, excitation, or chemical reactions. Each of these processes destroys a radiative property by which we can observe and identify the particle species, thus determening their lifetime or range in the coma. Each destruction process will, of course, give rise to one or more new particle species; e.g., H_2O, which has been detected by one of its microwave transitions (Jackson et al., 1975) will not not only dissociate into O, but also into H which is detectable in the UV (see, e.g., Code et al., 1970; Bertaux and Blamont, 1970; Bless and Code, 1972; Jenkins and Wingert, 1972; Bertaux et al., 1973; Carruthers et al., 1974; Feldman et al., 1974; Opal et al., 1974; Bohlin et al., 1975; Broadfoot et al., 1975; Page, 1975; Keller et al. 1975), and OH which has been detected in the radio (Biraud et al., 1974; Turner, 1974), infrared (Meisel and Berg, 1974), visible, and UV (Blamont and Festou, 1974; Feldman et al., 1974) ranges of the spectrum. Detection depends strongly on the

oscillator strength of the transition, the excitation mechanism (e.g., the intensity of the solar spectrum in the wavelength range of the transition), and on the species abundance. Some radicals may be abundant because they are produced by several different parent or mother molecules.

The only true (material) destruction mechanism of the coma--i.e., the limit to the approximately radial expansion of the gas and dust--is determined by the interaction of the coma with the solar radiation field and the solar wind which sweep the material into the tail.

5. COMA: STRUCTURE

Phenomenologically the gas coma can be categorized by three major parts that fit the observations of most comets:* (1) the inner, molecular, chemical, or photochemical coma, (2) the visible or radical coma, and (3) the atomic or UV coma. The absolute and relative ranges of these depend on heliocentric distance. At 1 AU typical values are $\sim 10^4$ km for the molecular coma, several times 10^5 km for the radical coma, and a maximum of $\sim 10^7$ km for the atomic coma. Table 2 summarizes the observational data of recently identified species. More complete information can be found in the compilations of Woszczyk (1962a), Richter (1963) and Arpigny (1972). The frozen gases in Whipple's icy conglomerate model of the nucleus must consist primarily of molecules containing the elements H, C, N, and O. Sulfur compounds, although not yet spectroscopically identified in the coma are very likely to occur. Other cosmically abundant elements such as the noble gases He and Ne, because of their

*Exceptions may be very small or nearly depleted comets.

low latent heat of vaporization, have probably not been trapped during formation of comets or have since escaped. The elements Na through Si and K through Ni are most likely associated with the more refractor grains. Sodium, one of the more volatile of these elements will receive more attention in the discussion on dust tails.

Some spectral lines which have been observed but not identified are listed in Table 3. Molecules, radicals, or atoms not found in special spectral line searches are listed in Table 4. If a species has been identified in some other spectral region then it is not included in this table.

Most direct deductions about the nucleus can be made from observations of the inner or molecular coma. Unfortunately several complications exist: (1) Most observations of molecular spectra must be made in the radio and infrared regions of the spectrum. (2) The size of the molecular coma at 1 AU geocentric distance is only 0.5 min arc, quite difficult for daytime observations with IR or radio telescopes. (3) The solar spectrum is weak in this region and excitation of the molecules must occur through collisions or through radiative pumping (see, e.g., Biraud et al., 1974; Mies, 1974). Besides H_2O, which may be very abundant, the only other mother molecules detected so far are HCN and CH_3CN, i.e., molecules with the strongest line transitions (Huebner, 1971). Radio observations of molecules were recently reviewed by Snyder (1975). Results from infrared spectroscopy have generally been too marginal for positive identification of any gas phase molecular transitions

(O'Dell, 1971a; Barbieri et al., 1974b), but in C/Kohoutek (1973 XII) Meisel and Berg (1974) identified CN and OH at $\lambda = 1.1\ \mu m$.

The range of the molecular coma is determined by the mean lifetime, τ, of the molecules in the field of solar radiation, f_ν,

$$\tau = \left[\int \left(f_\nu / r_h^2\right) \sigma_\nu \, d\nu\right]^{-1} . \tag{5}$$

Here σ_ν is the total cross section for ionization and dissociation (including predissociation and autoionization). Collisional ionization and dissociation would reduce the lifetime, but these effects are usually too small (Vanýsek, 1969a). However, they are important for sun grazing comets (Spinrad and Miner, 1968). From the data of Potter and Del Duca (1964) and Stief et al. (1965), and Stief (1966), typical ranges, τv, are 10^4 to 10^5 km. These are larger than the range of mother molecules obtained from the analysis of coma observations. Taking into account solar emission lines and predissociation, Jackson (1975) finds better agreement with the observed values. The range of mother molecules also approximately coincides with the range for which the fluid dynamical model is valid in large comets, i.e., the range for which molecular collisions are important. The range of radicals, R_{coll}, in the coma at which the mean free path for collisions, d_{coll}, is equal to R_{coll} is

$$R_{coll} \doteq d_{coll} = \frac{1}{n(R_{coll})\ \sigma_{coll}} = \frac{4\pi\ v\ R_{coll}^2}{Q\ \sigma_{coll}} \quad , \tag{6}$$

or, assuming a gas production yield of $Q \approx 10^{30}\ s^{-1}$, $v \approx 3 \times 10^4\ cm\ s^{-1}$, and $\sigma_{coll} \approx 10^{-15}\ cm^2$ for molecule-molecule collisions, $R_{coll} \approx 3 \times 10^4$ km. A more refined calculation by Jackson and Donn (1966) gives an equivalent result. Thus chemical reactions may have a strong influence on the composition of the coma of brighter comets or those of small q, (see, e.g., Donn and Urey, 1957; Jackson and Donn, 1968; Biermann and Diercksen, 1974).

Recently Oppenheimer (1974) has reinvestigated the gas phase chemistry in the coma. He finds that the reaction rates involving charged radicals similar to the ones proposed for production of interstellar molecules can give results consistent with observations. A detailed calculation taking into account Oppenheimer's model for the chemical reactions in conjunction with reactions on grain surfaces, solar radiation, fluid dynamics, and observational comet data should yield new clues about still undetected molecular and radical constituents in the coma and the composition of the nucleus.

Expanding the models presented by Jackson and Donn (1966) and Dolginov and Gnedin (1966), Shul'man (1969a), developed a single-component fluid dynamic model with spherical symmetry. In a second paper he describes the nucleus-coma boundary layer (Shul'man, 1969b). In his

model the conservation equations for mass, momentum and energy are solved for the adiabatic and nonadiabatic cases and he discusses the general solutions under various different conditions for subsonic and supersonic flow and shock discontinuities. A specific solution for the nonadiabatic case can be obtained only if the rate for heat input is known as a function of distance from the nucleus. This was modeled by Wallis (1974), for the case that water dominates the chemical composition of the coma. He finds that the flow is supersonic if the heating is uniform throughout the coma, but if collisions with dust dominate in the inner coma then the flow is subsonic going over into supersonic with increasing distance from the nucleus.

Many-component fluid dynamic models were developed by Mendis et al., (1972) with a nucleus as the single central source and by Ip and Mendis (1974) including an icy halo as an additional extended source. They simplified their approach by substituting a polytropic equation of state for the equation of energy conservation.

Although understanding the physical processes occurring in the molecular coma is a prerequisite for modeling the outer parts of the coma and its interaction with the interplanetary medium and for deducing information about the nucleus, present models are too crude to be useful for detailed predictions. They are, however, a good start for developing a model that is internally selfconsistent. The paucity of good, fundamental atomic and molecular data places severe restrictions on the successful development and application of coma models.

The radical coma is richest in spectroscopic data. It is here and in the atomic coma that application of the microscopic material properties are most useful. The detailed frequency dependence of the solar spectrum, i.e., the Fraunhofer lines, play an important role in the radiative transfer by coma radicals. A beautiful illustration of this has been given again recently by Arpigny (1972). Doppler shifts due to the relative motion of the comet with respect to the sun (Swings, 1941) and due to the differential expansion velocity of the coma (Greenstein, 1958) are the cause for the anomalous behavior of high resolution rotational line intensities. The resonance fluorescent character of the visual coma emissions is satisfactorily explained for CN and the diatomic hydrides (Swings, 1965; Arpigny, 1972). For C_2 the fluorescence mechanism involves the $^3\Pi_u$ metastable state for the lower electronic level rather than the ground state $^1\Sigma_g^+$. Wurm (1963) explained the high temperature (2500 to 5000°K) distribution of vibrational and rotational states of C_2 to be due to the absence of allowed vibrational or rotational transitions in the electronic ground state of homonuclear molecules. Herzberg (1975) has investigated the spectrum of C_3 in terms of its very low lying bending frequency (64 cm^{-1}) in the ground state. The validity of the fluorescence mechanism for NH_2 has been demonstrated by Woszczyk (1962b). Note that NH_2 is isoelectronic with H_2O^+ which has been identified in comets recently. We will discuss the spectrum of H_2O^+ with that of other comet tail ions. The O spectrum, which in the visible is not due to resonance fluorescence is discussed with the other atoms in the coma.

Hydroxyl is so far the only radical that has been observed in the ultraviolet, visible, infrared, and radio range of the spectrum. The UV observations have been reviewed recently by Keller (1975). UV observations of several comets indicate that the production rate of H is about twice that for OH. Deviations from this factor of two appear to lie within the error limits of the observations. Since OH decays further the result is consistent with the assumption that OH and H stem from the same mother molecule: H_2O.

The abundance ratio $^{12}C/^{13}C$ has been deduced from the relative brightness of isotopes of C_2 in several comets. The ratios presented in Table 5 are in reasonable agreement with terrestrial values. The $^{12}C^{13}C$ A→X 1-0 transition, on which the ratios are based, are blended with NH_2 lines. The value quoted for C/Ikeya (1963 I) is probably low due to an incorrect value for the NH_2 blending ratio (Owen, 1972). Because of the blending with the NH_2 lines it is unlikely that the accuracy of the $^{12}C/^{13}C$ ratio will ever be good enough to be useful as a discriminant for theories on the origin of comets.

Prediction of production rates of radicals depends not only on the adopted model for the coma but also on the lifetimes of the mother molecule, τ_M, and its daughter (radical), τ_D. Because of the symmetry of the two lifetimes in the equation for particle density in an unmodified isotropic expansion model (Haser, 1957)

$$n(r) = n(R) \frac{v_M}{v_D} \left(\frac{R}{r}\right)^2 \frac{\tau_D v_D}{\tau_D v_D - \tau_M v_M} \left[\exp\left(-\frac{r-R}{\tau_M v_M}\right) - \exp\left(-\frac{r-R}{\tau_D v_D}\right)\right] \tag{7}$$

no unique result can be obtained. Table 6 summarized our knowledge of lifetimes as determined by solar radiation at 1 AU. It is not clear how collisions in the coma that affect dissociation, predissociation, ionization, autoionization, charge exchange and the corresponding inverse processes change the tabulated values. In a large, productive comet the radical coma falls in the intermediate range between fluid dynamic and free particle flow. Dolginov et al. (1971) have modeled the neutral molecule distribution taking into account dissociation lifetimes, optical depth, temperature, and acceleration in the solar radiation field to obtain information about physical parameters near the nucleus from the brightness distribution of C_2 and CN.

The molecular and radical coma also overlap with the dust coma. Models to explain various observed properties have been developed. The icy grain coma, discussed above, is one particular example. The reduction of the grain size through sublimation not only affects the motion of the grains but also the reflection (Mie scattering) of the solar radiation as a function of wavelength. For sun grazing comets like Ikeya-Seki (1965 VIII) even the refractory grains are sublimated (Huebner, 1970) giving rise to the atomic spectra of ele-

ments Na, K, Ca, Cr, Co, Mn, Fe, Ni, and Cu (Preston, 1967; Slaughter, 1969). The effects of heat exchange between molecules and grains were pointed out by Wallis (1974). Dolginov (1967) argues that certain grains, particularly carbon grains, can be condensed in the coma from carbon atoms which derive their existence from dissociation of volatile molecules containing C. The most successful model for the dust coma was developed by Probstein (1969). He solves the conservation equations for mass, momentum, and energy for the two-component fluid

dynamic model in the adiabatic case, taking into account the momentum and energy transfer terms in the dust-gas interaction. His solutions are expressed in terms of two similarity parameters, one of these is characterized by the particle size and density, the other by the mass flow rates of dust and gas. Nonadiabatic effects, chemical reactions, and dust-dust collisions are neglected; further he assumes that all dust grains are spherical and of the same size and do not contribute to the pressure, nor do they sublimate. He finds that dust grains attain their final velocity in a range of about 100 km from the nucleus.

The infrared continuum emission from comets was investigated by Krishna Swamy and Donn (1968). Their paper also contains a brief summary of infrared comet observation at that time. They assume the grains to be in radiative equilibrium so that

$$F_{abs}(a) = F_{em}(a,T) \quad , \tag{8}$$

where for absorption

$$F_{abs}(a) = \int_0^\infty F_\odot(\lambda) \left(\frac{R_\odot}{r_h}\right)^2 \eta(a,\lambda)\pi a^2 d\lambda \quad . \tag{8a}$$

and for emission

$$F_{em}(a,T) = \int_0^\infty \pi B(\lambda,T)\eta(a,\lambda)4\pi a^2 d\lambda \quad . \tag{8b}$$

Here a is the grain particle radius, $F_\odot(\lambda)$ is the incident solar flux at wavelength λ, $\eta(a,\lambda)$ is the absorption efficiency, and $B(\lambda,T)$ is the Planck function corresponding to the grain temperature T. Since grains cannot radiate effectively for $\lambda >> a$--i.e., the emissivity $\eta(a,\lambda)\ B(\lambda,T) < B(\lambda,T)$--the black body curve that best fits the observations corresponds to a temperature that is higher than would be predicted from the solar radiation at given heliocentric distance. This is typical for comet grains (see, e.g., Becklin and Westphal, 1966, Máas et al., 1970; Kleinman et al., 1971; Gatley et al., 1974; Ney, 1974a,b), indicating that they have a size distribution that peaks at one tenth to several tenth μm. An analysis of three comets by O'Dell (1971b) indicates that the particle radius peaks at $a \approx 0.1$ μm and the emissivity of the grains is about 0.7 and decreases in the infrared. He also showed that the average optical albedo, A, can be obtained from the infrared surface brightness, S_{ir}, and optical surface brightness, $S_{op}(\lambda)$,

$$\frac{A}{1-A} = \frac{F_\odot\ S_{op}(\lambda)}{F_\odot(\lambda)\ S_{ir}} \quad , \tag{9}$$

where $F_{\odot}$ is the wavelength integral of $F_{\odot}(\lambda)$. From this he found $A \approx 0.3$. Ney (1974b) finds $0.1 < a < 1$ μm, and $A \approx 0.1$ to 0.2.

The work of Ney (1974b) shows particularly clearly the onset of the scattered sunlight at $\lambda \approx 2$ μm and going to shorter wavelengths, and the grain emmission at longer wavelengths for a number of heliocentric distances between r = .15 AU and ~1 AU. The 10.6 μm silicate feature is displayed especially beautifully and the 18 μm features are apparent.

Barbieri et al. (1974a) have analyzed their infrared data to estimate the dust production. Expressed in terms of gas production rates, they find dust production in C/Kohoutek (1973XII) is 0.1 before perihelion and ~1 after perihelion. Noguchi et al. (1974) have made the first infrared polarization measurements on a comet. They find no strong wavelength dependence, the polarization is about 15 to 20% perpendicular to the tail direction and about of the same order of magnitude as in the optical region.

Preliminary results indicate that observations in the mm and cm wavelength continuum (Bruston et al., 1974; Hobbs et al., 1975) will develop into useful tools for comet research.

The basic models for free particle flow are the exospheric, isotropic expansion model and the fountain model developed by Eddington (1910) in which the molecules are accelerated in the direction away from the sun after isotropic ejection from the nucleus. Haser (1957) has improved these models by including the effects caused by the finite life of the mother and daughter molecules. His density distribution for the isotropic expansion model is given in Eq. (7). Wallace and Miller (1958) have shown that for the fountain model with

isotropic and monokinetic ejection the isophotes are circular arcs centered at the nucleus independent of the angle of observation just as in the simple isotropic expansion model, but terminated on the intersection with the parabolic envelope. Wallace and Miller (1958) also investigated the effects due to dispersion of ejection velocities and due to anisotropic ejection but still assuming symmetry about the sun-comet axis. This model is appropriate for a very slowly rotating nucleus but does not take into account asymmetries caused by the vaporization lag expected for more rapid rotation as suggested by Whipple (1950). Optical depth effects for strong line transitions can be of importance in the denser regions of the coma and in the antisolar direction of the coma where the insolation at that wavelength has been attenuated (Arpigny, 1965). Finally the absolute and relative Doppler shift (Swings and Greenstein effects), line profiles, multiple scattering processes and differential effects of gravity, radiation, and lifetimes must be considered. This has been taken into account in a non-steady state model by Keller and Thomas (1975) for their analysis of the H coma as observed by Lα light. Their approach using syndynes, loci of particles of same solar repulsive force ejected at different times, is akin to the procedure used by Finson and Probstein (1968a,b) in their analysis of dust tails. Production rates of H, based on UV observations, depend on the absolute calibration of the instrument and on the model used to interpret the observational data.

From the production rates we can now estimate the total H_2O mass loss for a few comets. For C/Bennett (1970II) Keller and Lillie (1974) observed H (Lα) and OH (UV) from OAO-2 as the comet moved from r=0.72 to 1.25 AU. They found the production rates varied as $r^{-2.3}$, with values at 1 AU of 3.0×10^{29} and $5.4 \times 10^{29} s^{-1}$ for OH and H, respectively. This is consistent with H_2O as the parent molecule and $Q_{H_2O}=2.9 \times 10^{29} s^{-1}$ or an H_2O mass loss of 8.7×10^6 gm s^{-1} at 1 AU. An integration of this mass-loss rate over the entire orbit with $r^{-2.3}$ law leads to a total mass loss of 2.3×10^{14} gm of H_2O; or from r=1.5 to q to 1.5 AU, 1.7×10^{14} gm.

For C/Tago-Sato-Kosaka (1969IX), $Q_H \sim 8 \times 10^{29}$ s^{-1} as reduced to r=1 AU, which leads to a total mass loss of 1.5 times that for C/Bennett if the same $r^{-2.3}$ law is integrated.

For C/Kohoutek (1973XII) Carruthers et al. (1974) published some beautiful isodensitometer traces of the Lα coma. Blamont and Festou (1974), Keller (1975) and Huppler et al. (1975) observed H in Lα, while Wycoff and Wehinger (1975) observed H_2O^+, leading to the production rates of H_2O given in Table 7a, line 3, at the solar distances in line 2. The observed integrated brightness laws for the comet given in Table 7b, line 2 for the intervals in line 1 are from Jacchia's results (1974) with an assumed $r^{-2.72}$ law from r=0.3 AU to q and out to r-0.3 AU. This assumption gives the comet's magnitude as $+2^m_.2$

at r=o.3 AU where the three laws converge and $0^{m}.0$ at q=0.142 AU, a conservative estimate of the brightening. On the basis of these variations in production rates, the comparison of mass losses were made at r=0.3 AU, (Tab. 7a, last line) and the rate adopted as 2.0×10^{8} gm s^{-1}. The agreement among these four measures attests to soundness of the theory relating them. The loss rates in Table 7b, line 3 are formal statements of the adopted rate at r=0.3 AU, reduced to r=1 AU with the three laws of line 2. Integrations lead to the total H_2O mass losses given in Table 7b, line 4 for the ranges in r as indicated. The total, integrated over the entire orbit, becomes 1.0×10^{15} gm of H_2O, or 1 km^3, approximately 5 times that for C/Bennett and 3 times that for C/Sago-Sato-Kosaka.

Note that the mass loss rates of H_2O alone for these bright comets at r=1 AU are the order of 1×10^{7} gm s^{-1}. With the complete utilization of absorbed solar radiation and a total heat to vaporize of 670 cal gm^{-1} for H_2O, the corresponding spherical radius of such a comet with albedo = 0.3 becomes 2.1 km. Sekanina and Miller (1973), on the basis of the dust tail and other physical characteristics, derive a radius of 3 km for C/Bennett, generally consistent with H_2O as a major constituent. Because C/Kohoutek was considerably fainter (~6 times) than C/Bennett at r=1 AU on its way out, we may surmise that the nucleus of C/Kohoutek was considerably smaller than that of C/Bennett

and that before perihelion it lost material more rapidly per unit area. This could be a consequence of a loose aggregate of icy grains in its structure, for which there is other evidence, or possibly the result of a composition of material more volatile than H_2O.

The production rate of H_2O for P/Encke reduced to r=1 AU was observed (Keller, 1975) at $Q_H=3\times10^{27}\ s^{-1}$, some two orders of magnitude below those for the bright comets discussed above, consistent with a very much smaller sublimating area for an intrinsically much fainter old comet.

The OH bands (0-0 at 3090 Å and 1-1 at 3142 Å) yielded production rates of $Q_{OH} \sim 2\times10^{29} s^{-1}$ for C/Bennett and $\sim 1\times10^{28}\ s^{-1}$ for C/Kohoutek, reduced to 1 AU (Keller, 1975). In addition to the OH lines, Opal et al. (1974) and Feldman et al. (1974) also identified atomic C at 1657 and 1561 Å and O at 1304 and 1356 Å. They find the production rates of C and O to be commensurate with that for H_2O in C/Kohoutek. Reduced to r_h=1 AU, $Q_C \approx 6\times10^{27}\ s^{-1}$ and $Q_O \approx 2.4\times10^{28}\ s^{-1}$. Large production rates of O were already predicted by Biermann and Trefftz (1964) on the basis of the observed forbidden transitions in the optical. These authors concluded that the excitation for the forbidden lines must occur during photodissociation in forming atomic oxygen.

Thus there now exists very strong evidence that water ice is a major constituent of comets: H_2O has been detected in a

microwave transition in C/Bradfield (1974b); H_2O^+ has been identified in several comet spectra, H and OH are very abundant in constituents in several comets and about in the ratio 2:1. Delsemme and Rud (1973) argue that H_2O is so abundant that it controls the production mechanisms for the coma. However, it is well to keep in mind that there still exist a number of factors which need to be ascertained more precisely before one comes to the conclusion that H_2O is the predominant constituent. As Fig. 2 indicates, a volatile ice can under certain conditions with appropriate albedo and optical depth, e.g., debris, give relative changes in production rates which will be very difficult to distinguish from those of water ice. As noted earlier (Sect. 2), the determination of the area x albedo of the nucleus for distant comets may not be possible if they are surrounded by a reflecting ice or dust coma. Atomic oxygen was about twice as abundant as OH in C/Kohoutek. Atomic carbon was nearly as abundant as OH. The upper limits of some of the unobserved molecules listed in Table 4 are rather high because their transition probabilities are low.

If it turns out that H_2O is indeed the predominant constituent of comets, we would possess a very important clue about the chemistry and formation of comets.

6. STREAMERS AND ION TAILS

Much progress has been made on comet tails since the review by Brandt (1968). We have already mentioned interactions of solar radiation and solar wind plasma with coma constituents. When the interaction with a particular constituent is sufficiently strong so that the corresponding acceleration, $\dot{v}$, results in a velocity, $v = \dot{v}\tau$, (within the lifetime, τ, of the constituent) that is comparable to its radial streaming velocity in the coma, then the constituent's elongation in the coma becomes so large that it assumes a tail-like structure. Interactions between the solar wind and ions formed in the coma may also result in streamers. Streamers are the whisker-like structures emanating from the vicinity of the head; they are shorter than the tails and make acute angles with the tail axis.

Structures in ion tails have been interpreted as ion concentrations which are formed by fluctuations of the gas production, of the ionization rate, or of the solar wind. The structures can be observed moving outward in the tail with accelerations that cannot be brought into agreement with interaction due to radiation pressure. This led Biermann (1951) to predict the solar wind, years before any space probe was launched. Ion tails are still used as probes to gain information about the solar wind flux (e.g., Biermann, 1966; Biermann and Lüst, 1966; Brandt and Hardorp, 1970; Brandt and Heise, 1970; Brandt et al., 1972, 1973) and are of particular value when observed out of the ecliptic and at very

small and very large heliocentric distances where it is difficult and expensive to utilize man-made space probes (Biermann, 1965).

The study of formation of streamers and ion tails through interaction with the solar wind has been carried out primarily by the Munich group for many years (Biermann et al., 1967, 1974; Brosowski and Schmidt, 1967; Brosowski and Wegmann, 1973; Schmidt, 1975). The penetration of the solar wind into the coma is controlled by ionization processes. The area of interaction is approximately determined from the balance of mass flow, $\dot{m}_{SW}$, of the solar wind and, $\dot{m}_C$, of the coma

$$\dot{m}_{SW} = n_{SW} v_{SW} \pi r^2 (M_{SW}/N_o) \approx \dot{m}_C = QM/N_o \quad . \tag{10}$$

Here $n_{SW} \approx 10^1\ cm^{-3}$ is the particle density in the solar wind, $v_{SW} \approx 3 \cdot 10^7\ cm\ s^{-1}$ is its velocity, and $M_{SW} \approx 0.5$ is its mean molecular weight; $Q \approx 10^{30}\ s^{-1}$ is the molecular production rate of the comet, and $M \approx 18$is the mean molecular weight of the coma gas. N_o is Avogadro's number. The area of interaction will then have a radius of $r \approx 10^6$ km, which establishes the size of the flow pattern. Biermann and coworkers solve the hydrodynamic model taking into account the change in composition of the

plasma; this requires separate equations for the conservation of mass and particle number. The interaction of the solar wind with the coma produces two boundaries; the outer boundary is a weak shock front at about 10^6 km from the nucleus. This bow shock separates the supersonic flow of the solar wind from the subsonic (possibly turbulent) flow, loaded with molecular ions. The second boundary usually at less than 10^4 km is the contact surface which separates the purely cometary ions and stream lines from the "loaded" solar wind and the stream lines originating in the sun. Between the two boundaries the magnetic field that is coupled to the decelerating solar wind builds up until its strength overcomes the pressure of the coma ions. At that point it accelerates the coma ions into the tail. The calculational procedure has advanced from the early one-dimensional estimates to three-dimensional calculations (see, e.g., Biermann et al, 1967, 1974). Independent calculations by Wallis (1973) are in good agreement. A recent review emphasizing the effects due to changes in gas production rates was presented by Schmidt (1975). Dissociative recombination, small electron pressure, large solar wind pressure and large associated magnetic fields tend to bring the contact surface closer to the nucleus, while a large gas production rate by the comet tends to move it out. Wallis (1975) and Schmidt pointed out that collisional interaction between ions and electrons leads to cooling in the inner coma up to distances of 10^4 km from the nucleus; the cooling can be so rapid that the incoming plasma flow is monotonic without a contact discontinuity.

Application of the plasma dynamical processes to Lα observations of comets leads to consistent estimates of solar wind flux (Biermann et al., 1974). Schmidt (1975) has extended the model description to also discuss the flow field in the tail itself, the contact surface in the tail, the acceleration of the plasma flow in the tail, the magnetic field configuration in coma and tail, and the cause for envelopes, streamers, and filaments. The magnetic field needs to be incorporated into the model explicitly to make these descriptions more precise.

To clarify the physical processes involved in the creation of the ion structures, observations in the neighborhood of the nucleus are needed. Lüst and Schmidt (1968) investigated the effects of the solar wind and corona on the tail of C/Ikeya-Seki (1965 VIII) during perihelion passage. Observations of such sun grazing comets can yield much information about the solar corona if the composition of the tail is known. It is very likely that dust was vaporized, molecules dissociated and atoms ionized, but no spectra of the tail at time of perihelion are available. Spectroscopic (in the visual and UV range), infrared, spectrophotometric and polarimetric observations of tails of sun-grazing comets near perihelion may resolve questions about the composition of the grains and about the dynamics of dust tails.

Ion tails and perturbations on them have been observed and studied for many decades as, e.g., by Biermann (1951), Lüst (1962, 1967), Biermann and Lüst 1966), Brandt (1968),

Burlaga et al. (1973), Brandt et al. (1973), Hyder et al. (1974). Jockers and Lüst (1972, 1973) have analyzed structures in the tails of C/Bennett and C/Tago-Sato-Kosaka and conclude that some of the observed events are due to changes in the solar wind direction, and interplanetary shock waves. They also find that structures moved nearly parallel to the tail axis and velocities of the structures were significantly slower in the inner parts of the tail than on the outer regions. These observations are consistent with bulk motion of the plasma. Hyder et al. (1974) on the other hand conclude from their analysis of structures far out in the tail of C/Kohoutek that the observed helix and "swan" cloud do not represent bulk motion of plasma but are due to a wave motion with phase speed equal to the Alfvén speed of the tail plasma. Their interpretation requires a magnetic field in the comet tail with $10^2\gamma \leq B \leq 10^3\gamma$; this compares with the solar wind field of $B_{SW} \approx 20$ to 25γ at the same heliocentric distance (0.5 AU).

In the ion tail CO^+ is usually the most abundant constiuent. During observations of C/Kohoutek, Benvenuti and Wurm (1974) reported two unidentified doublet emissions. Since they were visible only in the tail direction it was very likely that they were ion emissions. Herbig (1973) reported three unidentified lines, two of these agreed with those reported by Benvenuti and Wurm. Herzberg and Lew (1974) compared the lines with the laboratory spectrum of H_2O^+ obtained by Lew and Heiber (1972) and on the basis of this tentatively identified the comet emissions to be

due to H_2O^+. A search in the recorded spectra of previous comets indicated that some of these lines had also been reported to occur in C/Ikeya (1963I) by Miller (1964) and in

C/Whipple-Bernasconi-Kulin (1942IV) by Swings et al. (1943). Observations reported by Wehinger et al. (1975) included many more lines which positively confirmed the identification of H_2O^+. The identified transitions are listed in Table 2. Discussions about intensity variations of H_2O^+ have been given by Benvenuti and Wurm (1975), Wyckoff and Wehinger (1975) and Wehinger and Wyckoff (1975). The latter also report detection of H_2O^+ in C/Bradfield (1974b). Production rates of H_2O based on H_2O^+ observations agree well with those based on UV observations of H and OH. Radio observations require further analysis as does the fluorescence mechanism for H_2O^+.

7. DUST TAILS

Dust tails are formed through interaction of solar radiation with individual grains. Only particles within a limited size range receive sufficient acceleration to move far into the tail. Even then their velocities are small compared to the solar wind speed and the velocity of ions in ion tails. The ratio of radial force due to solar radiation pressure, F_r, to the absolute value of force due to solar gravitational attraction, $|F_g|$, is expressed as

$$1 - \mu = \frac{1}{c} \int_0^\infty F_\odot(\lambda) \left(\frac{R_\odot}{r_h}\right)^2 \eta_p(\lambda,a)\ \pi a^2 d\lambda \Big/ \left(\frac{4\pi}{3r_h^2}\right) a^3 \rho G M_\odot$$

$$= \frac{3\ R_\odot^2}{4c\ GM_\odot\ a\rho} \int_0^\infty F_\odot(\lambda)\ \eta_p(\lambda,a) d\lambda = \frac{0.9137 \times 10^{-15}}{a\rho} \int_0^\infty F_\odot(\lambda)\eta_p(\lambda,a)d\lambda \quad . \tag{11}$$

Here $F_\odot(\lambda)$ is the wavelength dependent solar flux, $\eta_p(\lambda,a)$ is the Mie efficiency factor for radiation pressure, G is the gravitational constant, and $R_\odot$ and $M_\odot$ are the solar radius and mass. The efficiency factor η_p is a function of the wavelength-dependent refractive index of the grain material and becomes very small for dielectric particles with size $a < \lambda/(2\pi)$. The maximum of $F_\odot(\lambda)$ is at $\lambda \approx 0.46$ µm. Thus radiation pressure is small for dielectric particles with $a < 0.05$ µm. For metallic particles η_p does not fall off as rapidly with a as for dielectrics. Thus small metallic particles will be accelerated preferentially. This effect may be observable in comet tails. It should be noted that $1 - \mu$ is independent of heliocentric distance, but the maximum size of particles that can be lifted off the nuclear surface by the drag forces of the sublimating frozen gases (as discussed in Sect. 4) does depend on heliocentric distance. Large particles are also less efficiently accelerated than small particles as evidenced by the 1/a in Eq. 11. The net effect is that dust tails are made up primarily of particles in the size range $0.1 < a < 1$ µm.

Finson and Probstein (1968a,b) very successfully applied the theory of Probstein (1969) to the analysis of comet tails in terms of <u>syndynames</u>

(or syndynes) and synchrones, loci of bursts of particles with a large range in size. They concluded that the gas production needed in the head to produce the observed dust tails is in agreement with the newer high values as mentioned earlier (Sect. 3). Sekanina and Miller (1973) applied Finson and Probstein's model also to C/Bennett. They find a sharp peak in the size-density distribution at $a\rho = 5.6 \times 10^{-5}$ g cm^{-2} (probably assuming a constant value for $\eta_p = 1.5$). The gas production rate is consistent with the newer high values.

O'Dell (1974) points out the inaccuracies of the analysis for small particles in the tail. He compares the values obtained by Sekanina and Miller with results obtained from infrared observations and from optical scattering (Stokes, 1972) and concludes that the peak of the distribution occurs at $a\rho/\eta_p = 1.5$ to 5×10^{-5} g cm^{-2}.

Levin (1964) and Spinrad and Miner (1968) point out that the length of observed Na tails is inconsistent with the expected mean life-time of Na. Huebner (1970) and Sekanina (1974b) suggested that Na is chemically bound in compounds with latent heat of sublimation large enough to cause a slow release of Na, i.e., to increase the lifetime. Since the most likely Na compounds have a latent heat that is too small to account for the observed lifetime in comets, Huebner (1970) suggested that the Na is enclosed in the matrix of dust grains with high latent heat and it is the sublimation of these matrix-grains that controls the slow release of Na.

Sekanina (1973a, 1975) has argued for the existence of icy grains to explain the dynamics of observed tails of comets at large heliocentric distances (e.g., comets Baade, 1955 VI, and Haro-Chavira, 1956 I). His results depend very sensitively on the assumed, but not unreasonable, parameters. However, the important conclusion--independent of the composition of the grains--is, that comets must carry a reservoir of material that is more volatile than water to explain the observed coma and tail at heliocentric distances larger than 4 AU. This is consistent with the earlier conclusions by Huebner and Weigert (1966).

The occasional observation of an anti-tail--i.e., a tail apparently pointing in the sunward direction,was explained by Bredichin (1877) and Whipple (1957) as being due to an accumulation of cometary debris which is ejected in the plane of the comet's orbit and thus becomes visible when the earth moves through that plane. Sekanina (1973b) predicted that an anti-tail should be observed in C/Kohoutek; it was first observed by Gibson (1974)in Skylab and later analyzed by Keller (1975), Sekanina (1974b) and Gary and O'Dell (1974). Sekanina, applying the Finson-Probstein model, finds a substantial excess of large grains with size of order 0.1 to 1 mm. He also points out that the rate at which grains reduce in size due to sublimation caused by solar radiation plays an important role in explaining the dynamics of

the tail. Gary and O'Dell find $a\rho \approx 2 \times 10^{-3}$ g cm^{-2}, which is about a factor 10 smaller than Sekanina's value. These large grains were ejected from the nucleus a long time (from 2 to 8 months) before the observation of the anti-tail and moved with the nucleus in the comet's orbit. Grün, Hoffman and Kissel (1975) observed similar grains from C/Kohoutek directly in space from HEOS 2. Ejection appears to have occurred at $r>3.8$ AU, the grains having a radiation pressure/solar-gravity ratio ≈ 1.0. Ney (1974b) finds that the particles in the anti-tail do not show the silicate features at 10.6 and 18 μm contrary to the observations of the normal dust tail. He interprets this to large grains, $a > 10$ μm in the anti-tail, versus $a \approx 0.2$ to 2 μm in the normal dust tail.

8. PROBLEMS OF ORIGIN AND FUTURE STUDIES

Comets have clearly been formed in regions of low temperature (< 100 K) and have never been heated greatly, except possibly (by radioactivity?) in the central nuclei of very large comets. This possibility may possibly be proven if it can be shown that some of the near-Earth asteroids are actually the nuclei of old comets (from space-probe studies).

Comets have been stored since their formation (when?) at great solar distances in the Öpik-Oort cloud and may have gained an outer coating of volatile interstellar dust. The great activity at large

solar distances by some new comets support this view. Whether comets were originally formed in fragmented interstellar clouds (Whipple, 1951; McCrea, 1960; Cameron, 1962) or in the region of the outer planets (Kuiper, 1951; Whipple, 1951) may be demonstrated eventually by precise measurements of their elemental abundances or by further theoretical developments. As yet theory is not adequate to prove that fragmented interstellar clouds can condense into comets or that planetary and stellar perturbations in sequence can move the comets from the outer planetary region to the Öpik-Oort cloud. Öpik's (1932) original theory can be used to support the latter alternative; more recently (1966b) he finds it valid. Thus the outer planetary origin appeals to most investigators. No theory of recent ($<10^9$ yr) comet formation has proved acceptable as yet.

Low density material, which must consist chiefly of water (or ice) is continuously becoming more evident in solar system structure and history. The mean densities and structure of Uranus and Neptune (e.g. Ramsey, 1967) suggest that they are primarily composed of the easily freezable compounds of the solar mix, that is, comets or their near equivalent. Similarly the low densities of Jupiter's satellites, Ganymede and Callisto, as well as probably Saturn's Dione and Titan, attest to icy accumulation around the major planets. The carbonaceous chondrites of type 1 contain a significant quantity of water. Wetherill (1974) holds that a late bombardment of the Moon ($\sim 4.1 \times 10^9$ yrs ago) many have been

produced by comets or their counterparts. Whipple (1975) suggests that after the solar nebula dissipated, comets perturbed from the outer planetary region were so numerous that a cometary nebula formed within Jupiter's orbit, contributing a significant coating of volatiles on the Earth.

Definitive answers to these questions can be expected from experiments on space probes to comets. The technology for such research is well developed while an adequate supply of suitable comets is available. Cometary probes can give us fundamental measures of cometary structure, chemistry and abundances of elements while providing a space laboratory for plasma physics in the coma and the ion tail.

In the meantime ground-based and satellite observations of comets can be improved. The radio study of comets is in its infancy, having begun with C/Kohoutek, while infrared techniques are developing enormous power. The need for basic laboratory data has been emphasized throughout this paper. Laboratory simulations of comets as carried out, e.g., by Danielson and Kasai (1968) and Kaimakov and Sharkov (1969) can be very valuable experiments but their scope needs to be greatly expanded. Finally the physical theory of comets can be improved enormously to match more observations of greater variety and precision.

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TABLE 1

OBSERVED COMPOSITION OF COMETS*

Head:	H, C, C_2, C_3, CH, CN, $^{12}C^{13}C$, HCN, CH_3CN, NH, NH_2, O, OH, H_2O, Na, K, Ca, V, Cr, Mn, Fe, Co, Ni, Cu.
Tail:	CH^+, CO^+, CO_2^+, N_2^+, OH^+, H_2O^+, Ca^+. Continuum from particles including Silicate 10- and 18-μm bands in head and tail.

*For spectra see Atlas, Swings and Haser (no date)

Table 2

Recently Identified Cometary Spectra

Species	Transition	Wavelength	Comets
H	$3p\ ^2P^o \rightarrow 2s\ ^2S$	656.27 nm	1973 XII[HR]
C	$2p3s\ ^3P^o \rightarrow 2p^2\ ^3P$	165.7 nm	1973 XII[F,O]
	$2s2p^3\ ^3D^o \rightarrow 2s^2 2p^2\ ^3P$	156.1 nm	1973 XII[F]
O	$2p^3\ 3s\ ^3S^o \rightarrow 2p^4\ ^3P$	130.4 nm	1973 XII[F,O]
	$2p^3\ 3s\ ^5S^o \rightarrow 2p^4\ ^3P$	135.6 nm	1973 XII[O]
CH	$^2\Pi_{1/2}$, $J = 1/2$, $F = 1 \rightarrow 1$	90 mm	1973 XII[BC]
OH	$^2\Pi_{3/2}$, $J = 3/2$, $F = 2\text{-}2,\ 1\text{-}1$	180 mm	1973 XII[B,T]
	$X\ ^2\Pi\ 5\text{-}2$	1.08 μm	1973 XII[M]
CN	$A\ ^2\Pi_i \rightarrow X\ ^2\Sigma^+\ (0\text{-}0)$	1.1 μm	1973 XII[M]
H_2O	$J_{K_{-1}K_{+1}} = 6_{16} \rightarrow 5_{23}$	13.5 mm	1974b[J]
HCN	$J = 1 \rightarrow 0$	3.4 mm	1973 XII[H]
CH_3CN	$v_8 = 1$, $J_K = 6_3 \rightarrow 5_3$, $6_o - 5_o$	2.7 mm	1973 XII[U]
H_2O^+	$\tilde{A}\ ^2A_1 \rightarrow \tilde{X}\ ^2B_1$ $\left\{\begin{array}{l}(0,10,0\text{-}0,0,0)\Pi, \\ (0,9,0\text{-}0,0,0)\Sigma, \\ (0,9,0\text{-}0,0,0)\Delta, \\ (0,8,0\text{-}0,0,0)\Pi, \\ (0,7,0\text{-}0,0,0)\Sigma, \\ (0,7,0\text{-}0,0,0)\Delta, \\ (0,6,0\text{-}0,0,0)\Pi, \\ (0,5,0\text{-}0,0,0)\Sigma\end{array}\right\}$	548-754 nm	1963 I 1942 IV 1973 XII[BW,HL,W]

Table 2 (cont)

(BW) Benvenuti and Wurm (1974)

(BC) Black et al. (1974)

(B) Biraud et al. (1974)

(F) Feldman et al. (1974)

(H) Huebner et al. (1974)

(HL) Herzberg and Lew (1974)

(HR) Huppler et al. (1975)

(J) Jackson et al. (1975)

(M) Meisel and Berg (1974)

(O) Opal et al. (1974)

(T) Turner (1974)

(U) Ulich and Conklin (1974)

(W) Wehinger et al. (1975)

Table 3

Unidentified Cometary Microwave Spectral Lines

Frequency	Reference
8.18882 GHz	Giguere and Clark (1975)
86.2471 GHz	Huebner et al. (1975)
89.0105 GHz	Huebner et al. (1975)

Table 4

Molecules not found in Microwave Spectral Line Searches*

Radiating Species	Transition Searched		Frequency [GHz]
CO	$J = 1 \rightarrow 0$	(U)	115.2712
SiO	$v = 1,\ J = 2 \rightarrow 1$	(H)	86.2433
"HNC"	$J = 1 \rightarrow 0$	(H)	90.665
X-ogen (HCO^+)	$J = 1 \rightarrow 0$	(H)	89.189
NH_3	1,1	(C)	23.6945
	3,2	(A)	22.8342
H_2CO	$J_{K_{-1}K_{+1}} = 1_{11} \rightarrow 1_{10}$	(HS) (S)	4.8297
HNCO	$J_{K_{-1}K_{+1}} = 4_{04} \rightarrow 3_{03}$	(H)	87.9252
	$4_{13} \rightarrow 3_{12}$	(H)	88.2390
HC_3N	$J = 1 \rightarrow 0,\ F = 2 \rightarrow 1$	(G)	9.0983
	$10 \rightarrow 9$	(H)	90.979
	$2\nu_7,\ J = 1 \rightarrow 0,\ F = 2 \rightarrow 1$	(G)	9.1561
CH_3OH	$J_K = 4_2 \rightarrow 4_1$	(C)	24.9335
$CH_2(CN)_2$	$J_{K_{-1}K_{+1}} = 7_{16} \rightarrow 7_{07}$	(A)	23.0842
CH_3C_2H	$J_K = 5_0 \rightarrow 4_0$	(H)	85.4572
$(CH_3)_2O$	$J_{K_{-1}K_{+1}} = 2_{20} \rightarrow 2_{11}$	(H)	86.2229

*If a molecule, radical, or atom has been identified in a comet by any spectral line, then it is not included in this table.

(A) Avery and Andrew (1974)

(C) Churchwell et al. (1975)

(G) Giguere and Clark (1975)

(H) Huebner et al. (1975)

(HS) Huebner and Snyder (1970)

(S) Schröder et al. (1974)

(U) Ulich and Conklin (1974)

Table 5

Isotope Ratio of $^{12}C/^{13}C$

Terrestrial	89
Solar system	~90
Comets	
Ikeya (1963 I)	70 ± 15 (S)
Tago-Sato-Kosaka (1969 IX)	100 ± 20 (O)
Kohoutek (1973 XII)	115^{+30}_{-20} (D)
	135^{+65}_{-45} (D)

(D) Danks et al. (1974)

(O) Owen (1973)

(S) Stawikowski and Greenstein (1964)

Table 6

Lifetimes at r_h = 1 AU

Species*	τ [s]
H	2.2×10^6 (K) **
	1.3×10^6 (KT) **
OH	2×10^5 (KL) **
C_2	6×10^4 (D) **
CN	1.4×10^5 (D) **
HCN	9×10^4 (J)
$P(C_2)$	5×10^3 (V) **
	2×10^4 (D) **
P(CN)	1×10^4 (V) **
	5×10^4 (D) **
HC_2CN	$\geq 6.5 \times 10^4$ (P)
	1.3×10^4 (J)
$C_2(CN)_2$	$\geq 1 \times 10^5$ (P)
C_2N_2	$\geq 2 \times 10^5$ (P)
	1×10^4 (J)
CH_3CN	$\geq 3.4 \times 10^6$ (P)
CH_4	$> 1 \times 10^5$ (P)
$(CH_2)_2$	$> 1.5 \times 10^4$ (P)
$(CH)_2$	$> 1.5 \times 10^5$ (P)
	5.8×10^3 (J)
N_2H_4	2.5×10^3 (P)
CH_3NH_2	3.2×10^3 (P)

Table 6 (cont)

NH_3	1.5×10^4 (P)
H_2O_2	5.8×10^3 (P)
H_2O	7.3×10^4 (P)

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* P( ) stands for: parent of

(D) Delsemme and Moreau (1973)

(J) Jackson (1975)

(K) Keller (1973)

(KL) Keller and Lillie (1974)

(KT) Keller and Thomas (1975)

(P) Potter and Del Duca (1964)

(V) Vanýsek (1969)

** from comet observations

TABLE 7

a) Observed H_2O Loss for Comet Kohoutek (1970XII)

	B and F	W and W		Keller	Adopted
Observed	H, Lα	H_2O^+	H_2O^+	H, Lα	--
r (AU) at Obs.	0.60 out	0.90 in	0.72 out	0.43 out	--
Obs. loss rate (gm s^{-1})	$1.5x10^7$	$4.5x10^7$	$1.0x10^7$	$4.8x10^7$	--
*Loss (gm s^{-1}) at 0.3 AU	$1.92x10^8$	$4.8x10^8$	$2.5x10^8$	$1.80x10^8$	$2.0x10^8$

b) Total H_2O Loss

Region (AU from Sun)	$\infty > r > 0.3$	0.3-q-0.3	$0.3 < r < \infty$
H_2O Loss Law	$(r^{1.5+1.28r^{1/2}})^{-1}$	$r^{-2.72}$	$r^{-3.67}$
*H_2O Loss at 1 AU (gm s^{-1})	$1.41x10^7$	$7.6x10^6$	$2.4x10^6$
Total Loss (gm)	$0.15x10^{15}$	$0.80x10^{15}$	$0.07x10^{15}$

Total H_2O Loss entire orbit = $1.02x10^{15}$ gm

*calculated

Figure 1. Comet Kohoutek (1973 XII). Left: normal photograph. Right: in H, Lα, from Skylab, on same scale same day. Extent: some millions of kilometers. Courtesy National Aeronautics and Space Administration.

Fig. 1 . Comparison of sizes of different parts of a comet. Isophotes recorded in Lyman α radiation (dashed lines) indicate the extent of the hydrogen coma in comparison to isophotes of the head and ion tail (solid lines) observable in visible light. Knots in the tail and streamers near the head are not apparent in the coarse isophote presentation. Dust tails are usually shorter and curved. The sun is to the left.

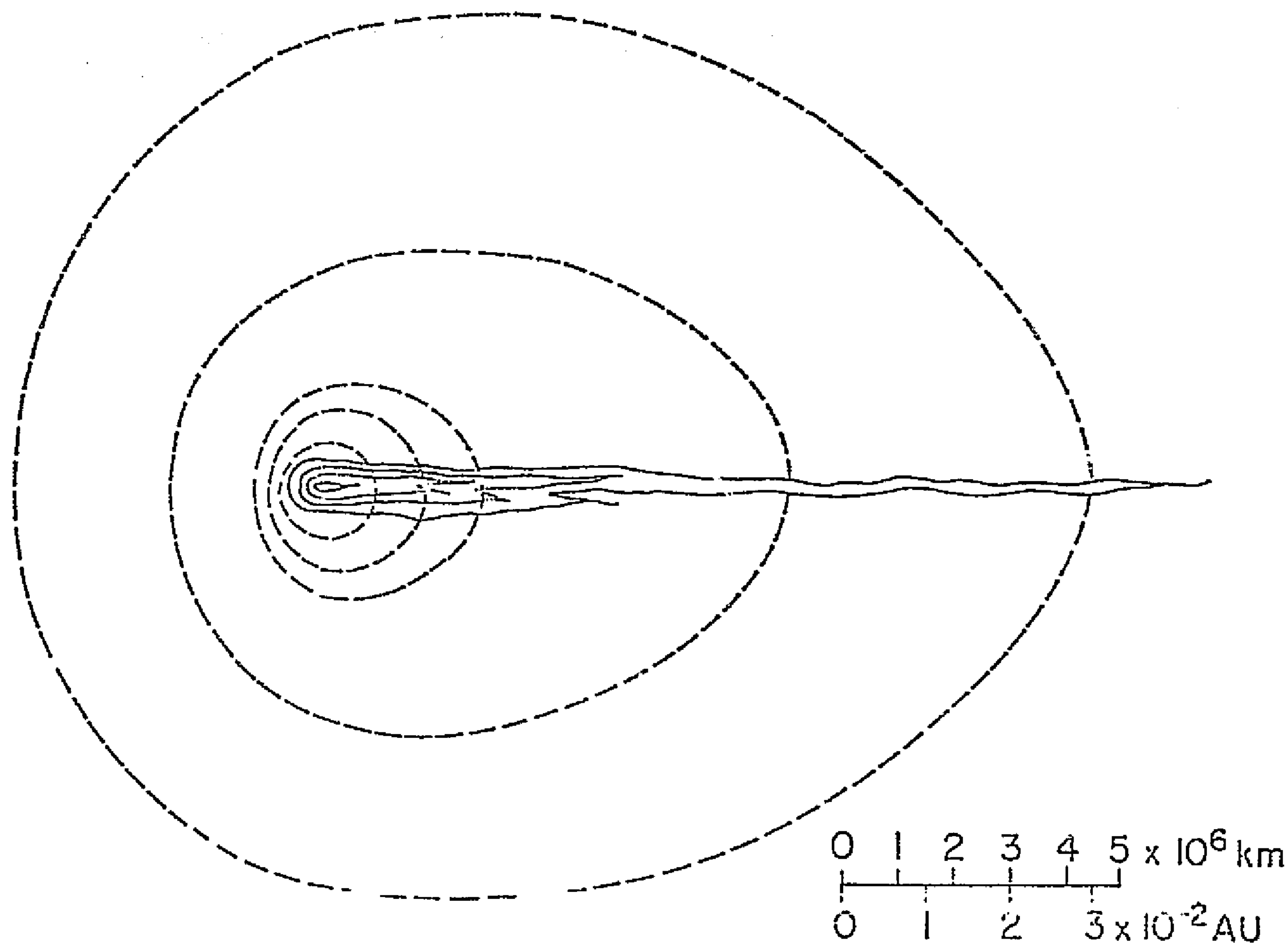
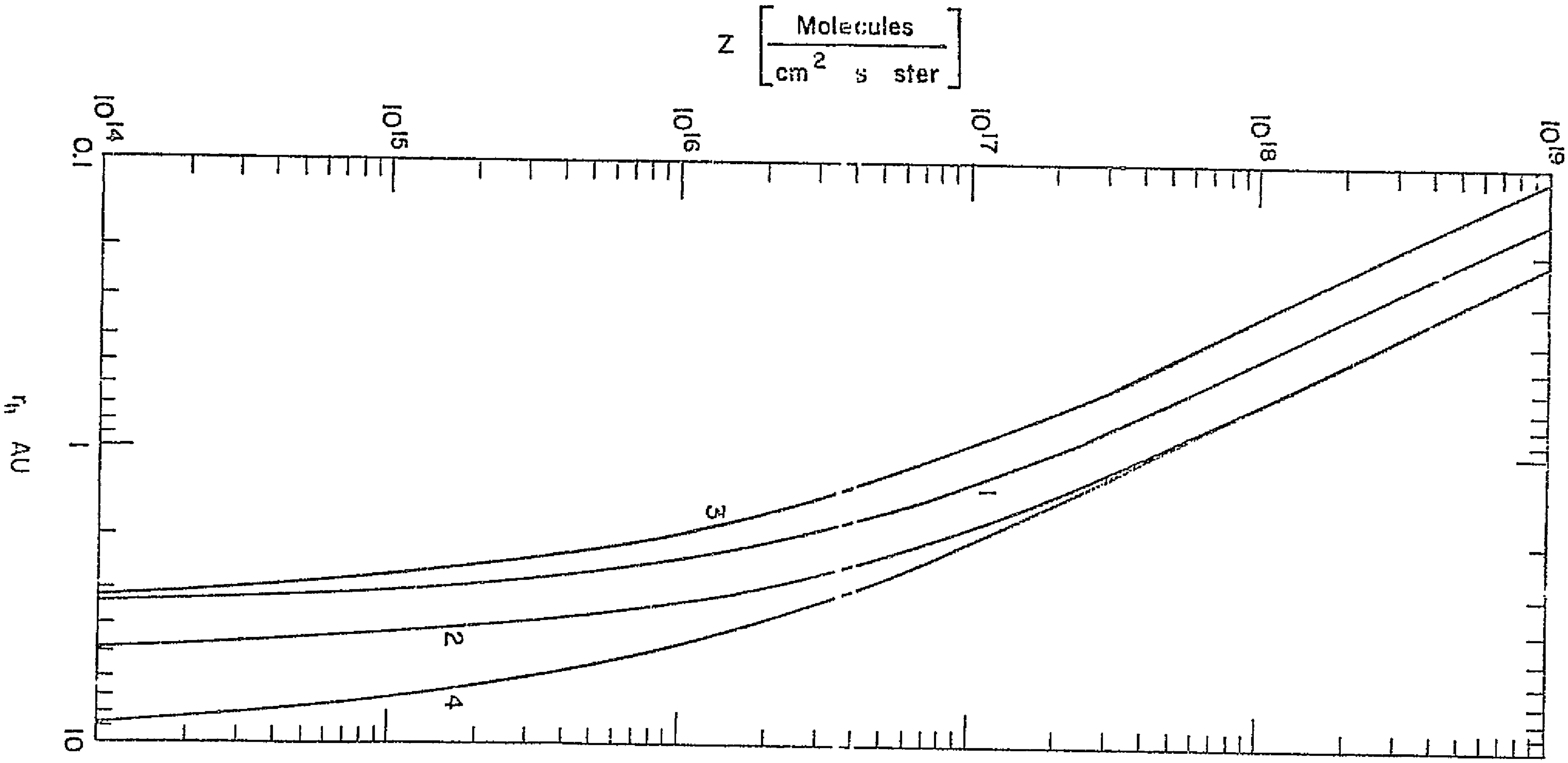

0 1 2 3 4 5 x 10^6 km
0 1 2 3 x 10^-2 AU

Fig. 2. Gas production rates, Z, vs. heliocentric distance, r_h, as obtained from Eqs. (1) and (2). For all four curves: $\kappa = 0$, $f_a = 0.9$ and $M = 18$. Curves 1 and 2 correspond to H_2O and clathrates ($L = 12$ kcal/mole, $\alpha = 7.03$) with $\varepsilon = 0.5$ and $(1 - A) \exp(-\tau) = 0.633$. Curve 1 is for a rapidly rotating nucleus with incident energy uniformly distributed over the entire surface, while curve 2 is for a slowly rotating nucleus with energy uniformly distributed over the sun-lit hemisphere only. Curves 3 and 4 correspond to a material with latent heat between CH_4 and NH_3 ($L = 6$ kcal/mole, $\alpha = 3.52$) with rapidly rotating nucleus. For curve 3 the nucleus is covered with debris such that $(1 - A) \exp(-\tau) = 0.156$, and $\varepsilon = 0.9$, while for curve 4 these quantities have the same values as for curve 1.


Z [Molecules / cm² s ster]
10¹⁴
10¹⁵
10¹⁶
10¹⁷
10¹⁸
10¹⁹
0.1
1
10
r_h AU
1
2
3
4

THE REALITY OF COMET GROUPS AND PAIRS

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ABSTRACT

Although the common genetic origin of the Kreutz family of sun-grazing comets has generally been accepted, there remains uncertainty with regard to the genetic identity among other groups of comets that have similar orbital elements. Porter has listed a number of such groups and Öpik has made a statistical study of the orbits of 472 comets with aphelion distance beyond Saturn. He lists 97 groups that show similarities among their three angular elements. He calculates an overall probability of some 10^{-39} that these similarities could have occurred by chance, and thus concludes that 60% or more of such comets fall into genetic groups containing from two to seven members.

This paper explores the statistical reality of Öpik's groups utilizing the Monte Carlo method of statistics as well as ordinary probability theory. The conclusion is reached that except for a few pairs the groups exhibit a similarity in their orbital elements that is no greater than might be expected by chance.

INTRODUCTION

The subject of comet families with common genetic origin was sparked by the recognition of the Kreutz (1888, 1891, 1901) sun-grazing family. Six or seven certain members of the family were discussed by Marsden (1967) and a more recent one has since appeared, Comet White-Ortiz-Bolelli (1970VI). The family clearly originated from the tidal disruption of a truly giant comet several thousands of years ago, with the probable subsequent splitting of some of the offspring. Öpik (1966) gives a good account of the physics of the process.

The splitting of comets by forces other than tidal disruption is relatively common; perhaps the most notable case is P/Biela. This can occur for "new" (see Oort and Schmidt), 1951, for definition) as well as for "old" comets at orbital positions far from perihelion or from disturbing planets (Whipple and Stefanik, 1966; Stefanik, 1966). Thus it seems natural to speculate that comets in similar orbits may have a common parent although survival of both components is not observed among such split comets. Porter (1952) lists 18 such groups and again (1963) 15 groups, not all in common, based on orbital similarities. Öpik (1971), in an exhaustive effort, intercompares the orbits of all the comets having aphelion beyond Saturn. By statistical methods he concludes that some sixty percent or more belong to families or groups, the largest groups containing six or seven members. The conclusion

demands that many comets in orbits of extremely long periods (millions of years) belong to groups with large enough memberships that two or more representatives come to perihelion in the span for which we have obtained good quality orbits. The implications with regard to the mode of origin of comets are startline and difficult to understand. Simple splitting with survival for both components is not adequate.

The purpose of the present paper is to explore the statistical reality of comet groups and pairs, excluding the sun-grazing family and the comets that have been observed to split. The Monte-Carlo method as well as ordinary probability theory will be employed and applied to the groups that Öpik and Porter have assembled.

THE MONTE CARLO METHOD APPLIED TO COMET GROUPS AND PAIRS

Because Öpik (1971) has made the most thorough attack on the problem of comet groups, including most of Porter's cases in his compilation, attention will be directed to Öpik's work. His method is to consider the orbits of 472 comets with aphelion distance $Q>10$ AU from the catalogue compiled by Baldet and de Obaldia (1952) and to segregate the comets into "boxes", usually 30° wide in the angular elements inclination, i, and longitude of the ascending node, Ω, reduced to the equinox of 1950.0. Thus there are primarily 72 boxes (6 in i: 0°-30°, 30°-60° etc.; and

12 in Ω similarly). In selecting comet groups, Öpik ignores the perihelion distance, q, on the grounds that q, next to Q, is the element most disturbed by planetary (or stellar) perturbations. The generic identification by means of q is thus lost quickly. This is true also for the eccentricity, e, and semimajor axis a, both of which are poorly determined or indeterminate among the nominally parabolic or nearly parabolic orbits providing most of the population considered. Öpik points out that $\Pi=\omega+\Omega$ ($\omega \equiv$ argument of perihelion) for orbits of $i<30°$ and $\Omega-\omega$ for $i>150°$ is much more stable than either ω or Ω, which can vary enormously.

With the third phase-independent angle, ω, Öpik selects his groups within the 72 specific boxes substituting $\omega+\Omega$ when $i<30°$ and $\Omega-\omega$ when $i>150°$. He finds 97 such groups allowing ω (or $\Omega\pm\omega$) to spread over two or more 30° intervals in some cases. A few of the groups encroach on adjacent boxes. He adds "supplementary" or "satellite" members (labeled by S) from outside the boxes where he concludes that the orbital similarities warrant inclusion. Note that the arbitrary limits on the boxes would be expected to exclude a number of legitimate members of groups.

To assess the reality of the groups, Öpik uses two types of probability criteria; first, the Bernoulli binomial probability of m members in a group with probability, p, individually, out of a population of n and secondly, a concentration criterion based

on apparent crowding of the group members in the boxes. On this basis he finds an overall probability of 3×10^{-39} that the groups as a whole result from random clumping of orbital elements. These probability criteria will be analyzed and revised in later sections of this paper as the applications are subject to some criticism. Because, in principle, probability criteria for groups that are selected opportunistically with somewhat variable criteria tend to be of doubtful validity, it seemed that some reasonably good criterion applied uniformly to a real and a random sample would provide a comparison from which the reality of the comet groupings could be judged. Hence the choice of a Monte Carlo approach to the problem.

In Öpik's study there are 107 comet orbits in the inclination range $60° < i < 90°$, from which he finds 17 groups, C21 to C33 and c86 to c89 in his Table 19, consisting of one group each with 6 and 5 members, 2 with 4 members, 6 with 3 and 7 with 2 (see Table 4, next section). The pairs are selected after the larger groups have been removed from the individual box. The closure ω ranges from 0° (for a pair) to 66° (the latter for the largest group). I selected the high-i set of orbits for a Monte-Carlo random comparison because the planetary perturbations on the angular elements of high-i orbits tend to be smaller than for low-i orbits, thus abetting the discovery of real genetic groups. Also

the Tisserand criterion for orbital identity is more sensitive to i for such orbits.

I derived the angular elements, ω, Ω and i by random numbers for the 107 orbits. Following the actual non-uniform distribution of the elements of the comet orbits, I doubled the probable number of Ω's in the intervals $30°<\Omega<150°$ and $210°<\Omega<300°$ in the random-number routine with respect to the remaining Ω's. Similarly, following Öpik's analysis, I set the probable number of ω's in the interval $0°<\omega<180°$ to be 0.62, vs. 0.38 for $180°<\omega<360°$. The attempt was only partially successful as 58 fell in the first range versus 49 in the second, i.e. ratios of 0.54 and 0.46 instead of 0.62 and 0.38. In ascribing values of q to the simulated comets, I distributed randomly the values of q for actual comets with $60°<i<90°$ from Marsden's catalogue (1975). For simplicity I considei . only parabolic orbits. The simulated comet elements are listed in the next section, Tables 3a and 3b, after having been segregated according to groups.

Within the 12 "boxes" in Ω (0°-30°, 30°-60°, etc.) I selected groups by the clustering of ω over much the same ranges chosen by Öpik. The goal was to find groups as large as possible within each box and then apportion the remainder in triplets or finally in pairs. The box score is presented in Table 1 comparing Öpik's groups with those of the simulated comets. Under the box headings

in Ω at the top, Table 1 lists in successive lines the number, n, in the box, the maximum number in the largest group, m_{max}, the corresponding, maximum range in $\Delta\omega$, then the number, m_2, in second largest group, its range $\Delta\omega$ and finally the numbers, m_i, in the ramaining groups. The simulated comets provide about the same number of larger groups (m=4,5,6,7), 5, as the real comets, 4, with the distribution of n among the boxes being somewhat comparable.

The largest group of simulated comets comprised 7, vs. 6 members compared respectively to the real comets, both out of boxes of 15 with a smaller $\Delta\omega$, 56° vs. 66°. The larger number of small groups and possibly the larger mean value $\Delta\omega$ for large groups among the simulated comets appears to be a matter of selection criteria. Real comets, however, appear to show a few extremely close fits among the pairs. We will return to this question later.

Grossly, then, the orbits of simulated comets appear to group in a manner similar to real comets. Let us now attempt to fine-tune the comparison utilizing probability criteria.

THE BERNOULLI FORMULA APPLIED

In calculating a criterion for the reality of his comet groups, Öpik applies the well-known Bernoulli formula for the probability that among a population of n members, exactly m will form a group for which the individual probability = p. He corrects the formula to include (approximately) the added chance for groups larger than m.

His formula for P_1 $(\geq m)$ is

$$P_1(\geq m) = \frac{n!}{m!\,m-n!}\, p^{m-1}\, \frac{(1-p)^{n-m}}{(1-q^*)} \quad , \tag{1}$$

where

$$q^* = \frac{(n-m)\,p}{(m+1)\,(1-p)} \quad . \tag{2}$$

The quantity $P_1(\geq m)q^*$ is the next term in the usual formulation of the binomial expression (without the $1-q^*$ term) so that the factor $(1-q^*)^{-1} = 1+q^*+q^{*2}+$etc., approximates the remainder of the series for small values of q^*. Also in Eq. 1 the inclusion of the factor p^{m-1} instead of the usual p^m, allows for the fact that the group may occur at any arbitrary point in the ω circle of 360°, increasing the probability by p^{-1}.

Because of the earlier mentioned arbitrariness in selecting the group memberships, m, and the value of $\Delta\omega$ for each group, such a theory has only limited significance as a probability criterion. But it can be used effectively to compare the significance of observed and randomly simulated groups selected in a uniform fashion.

In applying Eq. 1 to his cometary groups, Öpik, unfortunately, adopts $p=\Delta\omega/360°$, where $\Delta\omega$ represents the widest separation of ω in the Ω-i box. In forming pairs, to begin the process of group

formation, the first ω is arbitrary so that the target for the second is $2\Delta\omega$, not $\Delta\omega$; therefore, $p=2\omega/360°$ (or for integer degrees, $2\Delta\omega°+1°$ and $360p=2\omega°+1°$).

In forming triplets or larger clusters within the range $\Delta\omega$, the probabilities are linear for points around the circle. Thus the average separation for the first pair within $\Delta\omega$ will be $\Delta\omega/2$ making the average target $3\Delta\omega/2$. The average value for the total separation in the triplet will then be $3\Delta\omega/4$ so that the target areas for pairs, triplets, etc. are $\Delta\omega$ times 1+1, 1+1/2, 1+1/4, 1+1/8, etc. with the probability $p=\Delta\omega°/360°$, Eq. 1 being multiplied by the factor A, given by

$$A = \prod_{n=1}^{m-1} (1+2^{1-n}) \quad . \tag{3}$$

This multiplying factor will be used in subsequent calculations. Table 2 lists numerical values for A.

When whole degrees are used in ω or as in clustering of birth dates among a group of n people, a separation of one degree or day gives 3 possible target degrees or days for pairs and 7/3 for triplets. For large separations the values approximate those for the continuous case.

The term $(1-p)^{n-m}$ in Eq. 1 cannot be treated rigorously when $\Delta\omega$ is not specified in advance. A point falling near a group with separation $\Delta\omega$ might be chosen arbitrarily as a member of the group

increasing the separation. This effect is evidenced in Table 1 where the largest $\Delta\omega$'s occur for the largest groups. On the other hand, the effect is to increase the probability of larger groups. Thus I adopt $p=\Delta\omega°/360°$ to be used in the $(1-p)^{n-m}$ term of Eq. 1, providing a slight increase in probability.

A major question in Öpik's methodology concerns secondary groups in a box. His procedure is to eliminate the most "important" group from the box when dealing with the next "important" group etc. For example, suppose n=14 in the box, the most "important" group consisting of 5 members, $m_1=5$ separated by $\Delta\omega_1$. In calculating the probability criterion for the second group, he would adopt $n=14-5=9$ and $p=\Delta\omega_2/(360°-\Delta\omega_1)$, and progressively for successive groups. The above procedure can be justified if the groups are known to be physically real without "stragglers."

The Devil's Advocate (DA), however, does not admit <u>a priori</u> the conclusion of Öpik's paper, viz. that the groups are real physically. Denying this premise, the DA insists that the second group is a random sample from a population of n=14, not population n=9, and its expectation should be calculated independently of the existence of the first group. In comparing the Monte-Carlo comets with the real comets, it makes little difference which assumption is made. We have already seen, however, no strong evidence that the larger groups are real. Hence I choose to consider each group in a box independently of the others in calculating the expectation.

In summary: the expectation, E, for a group of m members with $p=\Delta\omega/360°$ from a box of n will be calculated from Eq. 1 with

$$E = AP_1(\geq m) \tag{4}$$

where A is given by Eq. 3 and q^* is obtained from Eq. 2 in which $p^*=p(1+2^{1-m})$ is substituted for p. The number n will not be reduced for any group within the box, each group being taken as an entity entirely independent of the others. In Tables 3 and 4 to follow, the values of ω, when small, are taken to 0°0. so that the discrepancy in A between integral and fractional degrees is eliminated. The actual values of $\Delta\omega$ are corrected for real comet orbits by Öpik's factors ϕ_1 and ϕ_2 given in his Table 18 and for the simulated comet orbits by the factor 1.08 for $\omega<180°$ and 0.92 for $\omega>180°$. This allows for the now uniform distribution of ω.

The Tisserand Criterion for ellipses is

$$TC = a_p/a + 2\sqrt{a(1-e^2)/a_p}\cos i \quad , \tag{5a}$$

and

$$TC = 2\sqrt{2q/a_p}\cos I \quad , \tag{5b}$$

Values of TC are included in Tables 3 and 4, where a_p is the semimajor axis of the perturbing planet (taken as Jupiter). The relevance of TC will be discussed in a later section.

In Tables 3a, b and 4, the first column, Gp., identifies the box ($\Delta\Omega$, Δi) by the whole number (or letter in Table 4) and the group by the decimal (or number in Table 4). The column, Fract., represents the number m in the group divided by the number, n, in the box. The angular elements are given to whole degrees but decimals were used in critical cases. The expectation, E, is calculated from Eq. 4 as described above. The Tisserand Criterion, TC, is given by Eq. 4 and 5 with Jupiter as the disturbing planet. The column ΔTC records the mean fractional absolute deviation from the mean TC_m in the group $\Delta TC = \Sigma|TC - TC_m|/TC_m$. Table 3b simply completes the record of the 107 random orbits. In Table 4, p, h or Q in the last column stand for parabolic, hyperbolic or oscillating aphelion distance, respectively.

Let us first discuss Table 3a to evaluate the significance of the probability criterion or the expectation, E. A number of pairs with large E are included in Table 3a. Within a box, E should predict the total number of such groups with $\Delta\omega$ less than or equal to that listed for the group. For example, Gp. 4.4 with $\Delta\omega=23°$ effective, has E=9.67, meaning that 10 such pairs should be expected in box 4, with $\Delta\omega \leq 23°$ for $\omega > 180°$ and $\Delta\omega = 23x(0.92/1.08)$ or $\Delta\omega = 19°$ for $\omega < 180°$. Actually 11 such pairs occur, 7 in Gp. 4.1, 2 in 4.2, 1 in 4.3 and 1 in 4.4.

Table 5 lists the pairs of high E for 10 groups in Table 3a. The $\Delta\omega$ limit observed is underlined for the ranges in ω as listed.

The last columns give the calculated E for the pair and the observed number, Obs., in the box, respectively. The expectation is underestimated by about 20 percent, the mean ratio E to Obs. being 0.81 instead of 1.0 for a perfect theory.

For larger groups the value of E is more seriously underestimated. For example, group 4.1, the largest with 7 members out of a box of 14, gives E=0.19. Correspondingly, Öpik's largest group, C21, with 6 members out of 15 gives E=0.25. His probability criterion $P_{.}(\geq m)$ is 0.065 for this group. We may conclude that the expectation, E, is an imperfect but useable criterion for comparing the real and the Monte Carlo simulation of comet orbits. Öpik's $P_1(\geq m)$ criterion severely underestimates the probabilities of his groups, often by a factor of 10 or more for secondary groups where the value of n has been reduced by the "most important" group.

APPLICATION OF THE EXPECTATION E CRITERION

Let us first examine the larger groups with m>3. Table 6 lists these groups for real comets, the headings following the pattern of earlier tables, except for $\overline{\Delta\Omega}$ and $\overline{\Delta i}$, which are the average differences between these elements within each group, to be discussed in a later section. The four C groups apply to the $60° < i < 90°$ boxes, the logarithmic mean of E being $\overline{E}_1 = \exp(\overline{\ln E}) = 0.50$. From Table 3a the four corresponding groups for random comets gives

E_1=0.48, practically identical with that for the real comets. If we now look at Öpik's complete tabulation we find 17 groups with m>3, which give a smaller value of $\bar{E}_1$=0.32±0.13.

A real problem arises in making a fair comparison of the real and random comets because of arbitrariness in selection of groups to be compared. Many of Öpik's groups would not have been listed had he been using Eq. 4 for E instead of his $P_1(\geq m)$. The answer to the question centers on the fact that we are really interested in the less probable groups because those with high expectation are presumably the result of chance, not genetically meaningful. So let us consider only groups with E<1.0. In this category Table 3a and 4 provide 12 random groups with $\bar{E}_1$=0.37±0.08 and 9 real groups with E_1=0.13±0.10. The comparison strongly suggests that the real groups are less probable than the random groups. But the numbers of groups compared are clearly too small to make the comparison definitive. Larger samples are required and emphasis should be placed on the less probable groups.

To provide a larger sample of Monte Carlo comets to compare with Öpik's complete compilation, I have set up 20 boxes, random in ω, with populations of n=15, 12, 10, and 6 each (860 in all), and have searched for the group (m members) that would give the smallest value of E in each box. The results are shown in Table 7, where m, Δω and E are indicated in successive columns for each value of n. In the boxes for n=15, the random of ω's were weighted

in favor of $\omega<180°$ and so $\Delta\omega$ is not strictly correlated with E. The distribution of E from the random groups is then somewhat comparable to Öpik's results in Table 8 where the group with the lowest value of E is listed for each of his published boxes. I have excluded from his list several groups that would have given $P_1(\geq m)>1.0$ on the basis of a constant value of n in each of his boxes (e.g. A5). Because of the minimum E restriction, the omitted groups with E<1.0 are: C23, E=0.78; C29, E=0.27; C32, E=0.42; b84, E=0.23; e94, E-0.75. Clearly these omissions do not bias the results negatively. No box was excluded from the Monte Carlo boxes.

Thus it is now possible to compare the least probable groups directly for sizeable populations of real and random comet orbits. This comparison appears in Table 9 where, for the groups in Col. 1 with the number of boxes in Col. 2, the logarithmic mean, $\overline{E}_1=\exp(\ln\overline{E})$, of the expectation and its standard deviation, σ, are listed in Cols. 3 and 4 respectively. From inspection of Tables 7, 8 and 9, Öpik's comet groups appear to be random clusters. The mean expectation $\overline{E}_1$ for the real comets, 0.33±0.07 agrees with that for the random comets, 0.28±0.04 well within the standard deviations. The comparison is a fair one in that some of Öpik's most probable groups have been excluded, while none were excluded from the random groups.

With this first demonstration of the random nature of the proposed real comet groups, we can explore further to find whether the conclusion is substantiated by other tests.

THE CONCENTRATION OF GROUPS IN BOXES

Öpik emphasizes the point that his comet groups are concentrated within the boxes, i.e. the members of the groups do not fill the boxes as fully as would be expected by chance. On this basis he calculates a second probability, p_2, based on the product rs where, for 30° boxes, $r=\Delta i/30°$ and $s=\Delta\Omega/30°$, Δi and $\Delta\Omega$ being the extreme spread of i and Ω for each group within the boxes. With this additional probability, $P_2=rs$, he reduces materially the total probability of his comet groups.

Now the spread of values of i and Ω within the boxes can better be considered from the vantage point of average distances between the observed values. For points at distance x along a line of unit length distributed proportionately to f(x), the average distance between points, $\overline{\Delta x}$, is given by

$$X\overline{\Delta x} = \int_{x=0}^{1} f(x) \int_{y=0}^{x} (x-y)\, f(y)\, dy\, dx + \int_{x=0}^{1} f(x) \int_{y=x}^{1} (y-x)\, f(y)\, dy\, dx \quad , \qquad (6)$$

where

$$X = \int_{x=0}^{1} f(x) \int_{y=0}^{x} f(y)\, dy\, dx + \int_{x=0}^{1} f(x) \int_{y=x}^{1} f(y)\, dy\, dx \quad , \qquad (6a)$$

in which y is taken as a moving second point within the unit length considered.

For uniformly distributed points $f(x)=f(y)=1$ and Eq. 6 integrates to $\overline{\Delta x}=1/3$. Usually then, we should expect the values of Ω in a group within 30° box to have an average separation $\overline{\Delta\Omega}=10^\circ.0$, if the values of Ω are randomly distributed. This is the average of all the absolute differences of the Ω values within the box, i.e., $m(m-1)/2$ differences. For integer degree values of the angles, the summation corresponding to Eq. 6 gives $\overline{\Delta x}=9^\circ.990$ in a 30° box.

For i, particularly in the intervals $0°<i<30°$ and $150°<i<180°$, the distribution function for comets is approximately, $f(i)=\sin i$. This leads to the result, in the interval i=0 to I, from Eq. 6 and 6a.

$$\overline{\Delta i} = I\,[\sin I(2+\cos I) - I(1+2\cos I)]/\sin I\,(1-\cos I)^2 \quad . \tag{7}$$

Evaluated for I=30°, Eq. 7 gives $\overline{\Delta i}=8^\circ.4045$. For the interval $30°<i<150°$, the sin i distribution function has no significant effect and practically $\overline{\Delta i}=10^\circ.0$ in the 30° boxes. From Tables 6 and 8 for Öpik's comet groups we can now evaluate the concentration within the boxes for i and Ω. The listed values of $\overline{\Delta\Omega}$ in the tables are corrected to 30° boxes. In taking mean values of $\overline{\Delta i}$ for $i<30°$ and $i>150°$ in Table 8, the tabulated value of $\overline{\Delta i}$ is

increased by the factor 10/8.40. Inspection of the B groups ($30<i<60°$) and the D groups ($90°<i<120°$) shows that the mean values of Δi are considerably smaller than 10°. Part of this effect arises from the fact that the original boxes are not uniformly filled. The groups derived from these boxes naturally give a spuriously small value of Δi. From actual cometary orbits I determined a correction to Δi of 1/0.85 for the B groups and 1/0.92 for the E groups, which corrections will be applied to Table 8 values of Δi in taking further means.

Table 10 displays mean values of $\overline{\Delta\Omega}$, $\overline{\Delta i}$ and $\overline{\Delta TC}$ with their standard deviations from Table 8, for various combinations of group sizes, m in the first column and expectation limits, E in the second column. The number of groups, No., appears in the third column.

For comparison with Table 10 I note that all of Öpik's larger groups ($m>3$) from Table 6, give mean $\overline{\Delta\Omega}=10\overset{\circ}{.}5\pm0\overset{\circ}{.}7$ and mean $\overline{\Delta i}=10\overset{\circ}{.}2\pm0\overset{\circ}{.}8$, completely consistent with a random selection in which $\overline{\Delta\Omega}=\overline{\Delta i}=10\overset{\circ}{.}0$. Öpik's probability p_2 is therefore equal to unity. For the larger groups we have a reaffirmation of the earlier comparison with the random comet elements via the expectation criterion. The larger groups represent random clustering in all respects both in Table 6 and in Table 9.

For the triple groups, $m=3$, the mean values of $\overline{\Delta\Omega}$ and $\overline{\Delta i}$ in Table 10 are consistent with random clustering as is the mean expectation, $\overline{E}_1=0.29\pm0.15$ (compare Table 9).

For orbits of small i and highly retrograde i, the values of Ω have a large range. Thus a fairer comparison might leave these orbits out of consideration. Among the 37 groups in Table 8 with $30° < i < 150°$, the mean value of $\overline{\Delta\Omega}$ is random, $10\overset{\circ}{.}4 \pm 1\overset{\circ}{.}0$.

For the 21 less probable groups with $E \leq 0.31$, there is possibly a significant decrease in mean $\overline{\Delta i}$, $7\overset{\circ}{.}8 \pm 0\overset{\circ}{.}8$ versus the $10\overset{\circ}{.}0$ expected from properly filled boxes. For the pairs we find the first conspicuous deviation from random clustering; the mean $\overline{\Delta i} = 5\overset{\circ}{.}3 \pm 1\overset{\circ}{.}2$ for the 9 pairs with $E \leq 0.31$. The slight reduction in mean $\overline{\Delta\Omega}$ for these pairs is not statistically significant however. From pure probability theory then, the data suggest that some of the pairs may represent genetically associated comets. Let us now look superficially into the dynamical background.

THE TISSERAND CRITERION

From the previous data including Table 10 we find no evidence for physical clustering of comet orbits in groups of three or more. It follows, therefore, that Öpik's concept of extensive clustering is not supported by the orbital data. We do not require some extraordinary theory of huge groups of comets with nearly identical initial orbits that, over millions of years, are still sufficiently concentrated to be recognizable within the 200-year span of our observations. The statistical evidence points to the possibility that a few real pairs are involved in our sample. These might arise from the normal splitting of comets. If so, the time

span of comet pair persistence need not cover a large number of revolutions about the Sun. In other words, the orbital characteristics within the pairs may not have altered greatly since splitting so that certain conservation laws can be applied. Specifically, because Jupiter is the major perturbing planet, Tisserand's criterion for the identity of comet orbits should apply to some extent.

As noted earlier, Öpik makes no use of the perihelion distance q in searching for or analysing comet groups. In fact, q is a moderately stable element, requiring a rather close approach to a major planet to change it much, or else a stellar perturbation. The latter demands a very large aphelion distance for effective change, unlikely because of the large random changes in 1/a produced by a passage through the inner solar system and also because nearby stellar passages are needed when the specific comet is near its presumed great aphelion passages. It might be justifiable to compare the q's in Öpik's groups to check their reality, but the Tisserand criterion provides perhaps a more significant quantity for comparison.

Although Tisserand's criterion is a special statement of the Jacobi intergral in the classical restricted three-body problem, it is also a conservation law applying to a close encounter of a small orbiting body with a massive planet (Whipple and Lecar, in preparation). The applicability of the criterion to comet pairs

is limited because multiple small perturbations by various planets could produce orbit changes that would change the numerical value of the criterion. Nevertheless, the criterion deserves some consideration. Numerical values from Eq. 5a, b appear as TC for individual random comets in Table 3a, b and for real comets in Table 4 along with the ΔTC, the mean absolute fractional deviation from the mean, for comet groups in these tables and in Table 8, of Öpik's minimum-E groups.

The mean value of ΔTC from the 33 random orbits of Table 3a can serve as a norm; the mean is ΔTC=0.38±0.04. For 8 groups of E≤0.5, ΔTC=0.41±0.07, indicating, predictably, no correlation with expectation E. From Table 8, Öpik's most favorable cases (min. E), the first three lines of Table 10 give the mean values of ΔTC as 0.29±0.03 for all 50 groups, 0.44±0.07 for groups of m>3 and 0.33±0.06 for groups of m=3. These values of ΔTC compare well enough with the mean random orbit value of 0.38±0.04 to support the other evidence for random clustering among real comets in groups of m≥3.

In Table 10 we notice a slight but statistically insignificant decrease in mean ΔTC for small values of E <u>except</u> for comet pairs. Here for the 9 pairs with E≤0.31, ΔTC=0.13±0.05 and for 4 pairs with E≤0.11 we see an even smaller value with small scatter, ΔTC=0.09±0.03. Thus eight pairs with small values of expectation also show small values of $\overline{\Delta I}$ and $\overline{\Delta TC}$, but not of $\overline{\Delta\Omega}$.

Do these unlikely pairs actually represent genetically associated comets? For pairs a82 and a83, involving relatively short-period comets, the answer to the question would involve an extensive analysis of the perturbation histories of the orbits, a task beyond the aims of this study. The reamining six pairs consist of nine parabolic orbits and three that were slightly hyperbolic near perihelion but elliptical at great solar distances before their observed apparitions (Marsden and Sekanina, private communication). The differences in q are in all cases significantly large, the minimum difference in a pair, $2(q_2-q_1)/(q_2+q_1)$ being 0.21 and the mean 0.49. A few of the parabolic orbits are undoubtedly poorly determined and might allow the possibility that the comets have made several, but probably not many, previous orbital revolutions. At least one (1955VI) of the three hyperbolic orbits appears to represent a first perihelion passage. Thus the orbits of the six nearly parabolic pairs do not appear to represent very ideal cases of splitting. Small initial orbital changes at splitting would have to be followed by large subsequent differential perturbations in q but not in the angular elements (stellar perturbations?). A comprehensive study of the possible perturbation histories of these pairs is needed. The result, however would not settle the question of possible genetic relationships for the poorly determined orbits. Several other close pairs can be found in Öpik's list among the larger groups ($m \geq 3$).

A remarkable case occurs in his group c88 (supplementary) in which all three angular elements agree within about 1° and the q's within 0.013 AU. The first, 1788II has a parabolic orbit, certainly not too well determined, and the second 1939VI, is P/Herschel-Rigollet with a period of 155 years. The Tisserand criteria do not agree quite so well, $\Delta TC=0.15$. But comet 1788II is the first observed apparition of P/Herschel-Rigollet!

Thus we are left with the possibility, still not proven, that a few genetically related comet pairs exist among the 472 orbits discussed by Öpik. The clustering in larger groups is statistically entirely random, except for the Kreutz group of sungrazers.

COMMENTS

Öpik notes in proof that "after 1965 and until the first half of 1971, there appeared 24 new comets according to the IAU circulars. Of these 13 happened to join one of the 97 groups." He calculates that this has a probability of "1/4000, or highly significant." The Monte Carlo group of 107 comets had, from Marsden's catalogue, 9 additions since 1966 for $60° < i < 90°$. Of these, 7 "joined" the groups of Table 3a and reduced the mean expectation E_1 of these 7 groups by a factor of 0.46, a combined probability of 0.004. It appears that real comets mix at least as well with simulated comets as with other real ones, to be expected if the clustering is a random phenomenon.

In view of the arbitrariness in comparing the real and random comets by means of the single least-probable group in each box, it is relevant to ask what would have resulted had, for example, the comparison been made on the basis of <u>all</u> groups with $E<1.0$? Such a comparison adds 5 earlier mentioned groups to the real comets and subtracts 10 from Table 8, leaving 45 groups which give $\bar{E}_1=0.24\pm0.5$. It adds 17 groups to the 92 random comets and subtracts 16, leaving 93 groups which give $\bar{E}_1=0.21\pm0.03$. Thus the comparison in $\bar{E}_1$ for $E<1.0$ provides convincing support for the random clustering of real comets.

Identifying comet pairs of common genetic origin, if such pairs exist, will require considerable effort. The method used by Öpik, similarity of angular elements, and by Porter, similarities in angular elements and also in the direction of the lines of apsides, must be supplemented by direct calculations of past motions in orbits of high quality. Various dynamical conservation laws involving q and eventually a and e as well as the angular elements must be employed. The present methods are only helpful in locating likely candidates and even then, more extensive search methods are required if true pairs are to be found and proven.

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Table 1

CLUSTERING AMONG COMET ORBITS ($60° < i < 90°$)

Ω Interval	0 30	30 60	60 90	90 120	120 150	150 180	180 210	210 240	240 270	270 300	300 330	330 360	Total avg
Real Comets (Öpik)													
n	3	11	15	13	7	3	8	6	10	12	12	7	107
$m_{max.}$	0	0	6	3	2	0	2	0	3	4	5	2	
$\Delta\omega$	-	-	66°	2°	4°	-	1°	-	9°	15°	35°	10°	18°
m_2	-	-	4	3	0	-	0	-	2	3	3	2	
$\Delta\omega$	-	-	30°	19°	-	-	-	-	0°	24°	2°	28°	17°
m_i	-	-	3	0	-	-	-	-	0	2	0	0	
Simulated Comets													
n	6	8	15	7	6	7	5	10	7	8	18	10	107
$m_{max.}$	2	3	7	3	2	3	3	2	2	4	5	5	
$\Delta\omega$	31°	27°	56°	17°	5°	32°	47°	7°	13°	48°	35°	34°	29°
m_2	2	2	3	2	2	2	2	2	2	2	4	3	
$\Delta\omega$	39°	5°	29°	8°	25°	5°	25°	7°	14°	16°	14°	35°	18°
m_i	0	2	2,2	0	0	2	0	2,2	0	2	3,3,2	0	

Table 2

FACTOR A

m	A
2	2.000
3	3.000
4	3.750
5	4.219
6	4.482
7	4.622
8	4.695
9	4.731
10	4.750
11	4.759
12	4.764
15	4.768
200	4.768

Table 3a

RANDOM COMET GROUPS

Gp.	Fract.	ω	Ω	i	q(AU)	E	TC	ΔTC
1.1	4/6	176°	7°	83°	0.653	0.20	0.127	0.33
		200	18	68	0.921		0.452	
		207	8	70	0.660		0.340	
		239	26	70	1.254		0.468	
2.1	2/8	40	39	74	0.342	2.76	0.201	0.14
		50	52	74	0.629		0.266	
2.2	2/8	109	54	64	0.811	2.24	0.491	0.54
		114	57	62	0.064		0.145	
2.3	3/8	293	47	90	1.037	0.63	0.066	1.01
		310	43	71	1.357		0.466	
		320	47	89	0.716		0.024	
3.1	2/7	16	60	65	0.444	0.95	0.350	0.14
		24	72	79	4.077		0.465	
3.2	3/7	176	84	64	0.723	0.18	0.464	0.75
		180	71	89	1.259		0.027	
		193	83	74	0.215		0.162	
4.1	7/15	14	92	61	0.499	0.19	0.429	0.34
		15	120	72	4.043		0.783	
		31	113	72	1.392		0.452	
		41	109	78	0.795		0.228	
		43	92	78	0.591		0.200	
		70	92	77	1.429		0.338	
		70	100	60	0.738		0.525	
4.2	3/15	172	109	78	0.473	4.43	0.185	0.28
		193	119	78	1.159		0.290	
		201	94	84	1.015		0.137	
4.3	2/15	232	117	64	1.719	8.86	0.705	0.06
		253	94	63	1.199		0.621	
4.4	2/15	287	91	82	0.498	9.67	0.125	0.62
		310	92	88	0.695		0.029	

Table 3a (cont)

Gp.	Fract.	ω	Ω	i	q(AU)	E	TC	ΔTC
5.1	2/6	180° 185	123° 127	63° 71	0.642 0.482	0.39	0.451 0.279	0.24
5.2	2/6	306 331	145 122	87 82	0.479 0.735	1.71	0.051 0.148	0.49
6.1	3/7	75 84 107	166 161 176	71 61 60	1.081 1.500 1.481	0.75	0.420 0.746 0.748	0.23
6.2	2/7	135 140	155 176	85 80	0.142 0.971	0.61	0.044 0.202	0.64
6.3	2/7	358 21	161 171	78 88	1.724 0.847	2.51	0.344 0.478	0.16
7.1	3/5	126 140 173	203 196 187	61 76 73	1.040 1.445 1.294	0.49	0.611 0.358 0.408	0.22
7.2	2/5	312 337	199 205	76 82	2.507 0.626	1.[illegible]7	0.478 0.145	0.53
8.1	2/10	30 48	210 228	66 85	2.079 0.752	4.07	0.741 0.092	0.78
8.2	2/10	101 113	229 226	64 86	0.061 1.104	2.85	0.134 0.095	0.17
8.3	2/10	259 266	220 220	78 70	0.625 1.100	1.50	0.199 0.455	0.39
8.4	2/10	301 308	240 238	77 61	0.764 0.006	1.50	0.240 0.047	0.67
9.1	2/7	61 85	259 256	87 80	0.902 1.234	1.60	0.064 0.242	0.58
9.2	2/7	275 288	240 268	4 5	0.822 1.786	1.29	0.302 0.429	0.17

Table 3a (cont)

Gp.	Fract.	ω	Ω	i	q(AU)	E	TC	ΔTC
10.1	2/8	103°	278°	78°	1.537	4.16	0.309	0.21
		133	289	65	0.843		0.478	
10.2	3/8	175	292	87	0.990	0.13	0.060	0.51
		182	297	88	1.009		0.037	
		186	286	90	0.958		0.008	
10.3	2/8	314	285	69	0.771	2.05	0.392	0.20
		330	293	87	0.716		0.586	
11.1	2/18	56	316	75	0.330	2.57	0.180	0.29
		59	319	84	0.534		0.098	
11.2	5/18	85	313	74	0.713	1.42	0.292	0.38
		94	330	79	0.629		0.189	
		105	317	75	0.795		0.286	
		114	306	82	1.009		0.165	
		120	312	88	0.776		0.044	
11.3	4/18	175	330	71	1.143	0.38	0.425	0.30
		181	302	78	0.646		0.202	
		183	301	79	1.756		0.322	
		189	315	80	0.970		0.204	
11.4	2/18	222	306	68	1.001	7.67	0.461	0.06
		234	315	72	1.102		0.411	
11.5	2/18	342	311	76	0.760	6.56	0.262	0.05
		352	325	65	0.307		0.288	
12.1	4/10	350	344	78	1.095	0.50	0.259	0.58
		6	346	66	3.747		0.992	
		12	334	61	1.647		0.781	
		24	333	85	4.276		0.215	
12.2	3/10	281	333	71	0.405	1.92	0.254	0.35
		294	334	60	0.176		0.258	
		316	331	70	1.537		0.531	

Table 3b

NON-MEMBERS OF GROUPS

Gp.	Fract.	ω	Ω	i	q(AU)	E	TC	ΔTC
1	3/6	36°	13°	76°	6.021		0.736	
		94	21	64	0.031		0.097	
		239	26	70	1.254		0.468	
2	1/8	260	39	75	0.801		0.295	
3	2/7	75	89	88	0.894		0.035	
		317	71	60	1:127		0.656	
4	1/15	134	113	62	1.702		0.760	
5	2/6	45	146	60	0.503		0.437	
		76	125	63	0.838		0.514	
8	2/10	163	231	80	1.171		0.244	
		213	237	64	0.420		0.350	
9	3/7	150	255	66	1.313		0.585	
		220	260	89	1.628		0.022	
		345	270	88	0.676		0.030	
10	1/8	223	283	67	4.051		0.979	
11	6/18	17	303	70	0.749		0.376	
		33	321	87	0.668		0.055	
		313	320	65	0.602		0.414	
12	3/10	85	341	63	0.707		0.468	
		165	337	67	0.960		0.475	
		208	354	64	0.293		0.293	

Table 4

ÖPIK COMET GROUPS (60°<i<90°)

Gp.	Fract.	Comet	ω	Ω	i	q(AU)	E	TC	ΔTC	p, h or Q
C21	6/15	1895III	299°	84°	76°	0.843	0.25	0.28	0.66	p
		1898VII	233	75	70	1.70		0.59		h
		1906II	276	73	83	0.723		0.14		p
		1948X	274	67	88	1.27		0.08		p
		1954XII	255	75	89	0.746		0.05		p
		1959VII	273	83	70	1.17		0.50		p
C22	4/15	1860III	77	86	79	0.293	2.74	0.14	0.62	p
		1827IX	47	78	85	0.176		0.06		p
		1956I	57	72	80	4.07		0.50		p
		1962IV	72	79	73	0.654		0.32		h
C23	3/15	1818II	112	72	90	1.20	0.78	0.03	0.69	p
		1879V	115	88	77	0.990		0.28		p
		1886II	120	69	84	0.479		0.10		h
C24	3/13	1863I	74	118	85	0.795	2.66	0.11	0.36	h
		1863VI	78	106	83	1.31		0.20		h
		1888III	59	102	74	0.902		0.32		19,600
C25	3/13	1850I	180	94	68	1.08	0.03	0.49	0.15	1883
		1878I	178	103	78	1.39		0.33		p
		1885II	178	93	81	2.51		0.36		h
C26	2/7	1853III	170	142	62	0.307	0.59	0.33	0.24	h
		1881II	174	127	78	0.591		0.20		p
C27	2/8	1774	137	183	83	1.43	0.31	0.185	0.01	h
		1840III	138	185	80	0.748		0.189		p
C28	2/8	1785I	206	266	70	1.14	0.003	0.45	0.15	p
		1898VI	206	260	70	0.626		0.34		p
C29	3/10	1863III	56	251	86	0.629	0.27	0.06	0.87	1360
		1880V	12	250	61	0.660		0.49		p
		1886VIII	32	259	86	1.48		0.08		p
C30	2/12	1883I	111	279	78	0.760	6.24	0.22	0.66	p
		1903IV	127	294	85	0.297		0.05		p
C31	4/12	1819II	13	276	81	0.342	0.22	0.10	0.62	p
		1899V	11	273	77	1.79		0.38		p
		1912II	26	298	80	0.716		0.17		2870
		1914III	14	271	71	3.75		0.78		h

Table 4 (cont)

Gp.	Fract.	Comet	ω	Ω	i	q(AU)	E	TC	ΔTC	p, h or Q
c87	3/12	1861II	330°	280°	85°	0.822	1.03	0.19	0.36	109.4
		1881III	354	272	63	0.769		0.52		360.6
		1889IV	346	287	66	1.04		0.52		911
C32	5/12	1810	115	311	64	0.970	0.42	0.53	0.72	p
		1857I	1[illegible]2	314	88	0.772		0.02		p
		1863V	116	306	64	0.771		0.45		p
		1933I	136	312	87	1.00		0.05		p
		1959I	101	323	61	1.63		0.74		p
C33	3/12	1729	10	314	77	4.05	0.03	0.51	0.52	p
		1900II	12	329	63	1.01		0.56		h
		1962III	11	304	65	0.031		0.09		p
c88	2/7	1932V	38	345	72	1.04	1.41	0.51	0.12	85.0
		1939VI	29	355	64	0.75		0.65		57.3
c89	2/7	1763	89	359	73	0.498	3.60	0.28	0.02	754
		1877III	117	347	77	1.01		0.29		971
c86	2/28	1846[illegible]	13	79*	85	0.664	6.37	0.38	0.17	35.2
		1935I	18	92	65	0.811		0.53		185.6

Table 5

EXPECTATION FOR SIMULATED PAIRS

Gp.	$\omega \leq 180°$ $\Delta\omega$	$\Delta\omega > 180°$ $\Delta\omega$	E	Obs.
2.1	9	10	2.76	3
3.1	8	9	0.95	2
4.4	19	23	9.67	11
5.2	21	25	1.71	2
6.3	22	24	2.51	3
7.2	21	25	1.17	2
8.1	18	21	4.07	4
9.1	24	28	1.60	2
10.1	30	35	4.16	5
11.4	10	12	7.67	11

Mean E/Obs.=0.81

Table 6

OPIK'S GROUPS WITH m>3

Gp.	Fract.	$\Delta\omega$	E	$\overline{\Delta\Omega}$	$\overline{\Delta i}$
A1	4/10	53*	1.89	14°.9#	11°.2
A4	7/12	62*	0.043	11.6+	10.2
C21	6/15	66	0.25	7.7	10.4
C22	4/15	30	2.74	7.2	6.2
C31	4/12	15	0.22	14.0	5.5
C32	5/12	35	0.42	7.4	15.4
D39	6/13	54	0.045	10.7	14.7
D45	4/9	47	1.24	11.5	5.0
E49	4/10	48	1.88	12.2	13.2
E56	5/12	33	0.28	6.2	8.8
E68	4/9	29	0.076	11.8	13.2
E71	7/15	37	0.061	6.1	6.9
E72	5/15	55	0.68	6.8	11.4
e92	4/10	42	1.38	11.0	14.3
F74	6/13	76*	0.48	10.3+	10.6
F76	4/6	60*	0.20	15.4+	9.7
F79	6/9	50*	0.047	13.0#	6.7

* $\Delta(\Omega \pm \omega)$

\+ Ω range = 60°

\# Ω range = 90°

Table 7

EXPECTATION E FOR RANDOM BOXES (Minimum E)

n=15			n=12			n=10			n=6		
m	Δω	E	m	Δω	E	m	Δω	E	m	Δω	E
7	64	0.25	6	61	0.23	5	60	0.40	4	69	0.29
7	62	0.22	5	50	0.55	5	56	0.32	4	66	0.26
7	55	0.12	5	38	0.23	5	49	0.20	4	50	0.12
7	22	0.01	5	37	0.20	5	45	0.15	4	14	0.003
5	57	1.50	5	36	0.19	5	44	0.14	3	48	0.82
5	51	1.57	5	31	0.11	4	65	2.02	3	47	0.79
5	36	1.31	4	25	0.40	4	47	0.95	3	41	0.62
5	47	1.24	4	25	0.40	4	43	0.77	3	34	0.44
5	44	1.02	3	23	1.85	4	37	0.53	3	26	0.27
5	40	0.76	3	8	0.28	3	12	0.35	3	26	0.27
5	33	0.42	3	3	0.05	3	11	0.27	3	19	0.15
4	35	1.25	3	4	0.08	3	10	0.24	3	8	0.03
4	27	1.15	2	1.6	0.56	2	7	1.63	3	7	0.02
4	35	0.82	2	1.2	0.44	2	6	1.41	2	36	2.57
4	17	0.36	2	0.5	0.19	2	1.3	0.32	2	17	1.30
4	14	0.22	2	0.4	0.16	2	1.2	0.30	2	15	1.16
4	7	0.03	2	0.3	0.10	2	0.27	0.07	2	11	0.87
2	1.2	0.37	2	0.2	0.08	2	0.24	0.06	2	11	0.87
2	0.8	0.07	2	0.2	0.08	2	0.18	0.04	2	5	0.41
2	0.01	0.01	2	0.01	0.003	2	0.18	0.04	2	3	0.25

Table 8

ÖPIK OPTIMUM GROUPS

(Minimum E)

Gp.	Fract.	$\Delta\omega$	E	$\Delta\Omega$	Δi	ΔTC	Gp.	Fract.	$\Delta\omega$	E	$\Delta\Omega$	Δi	ΔTC
A1	4/10	53*	1.89	14.9#	11.2	0.33	D34	2/10	3	0.95	2.0	3.0	0.15
A2	2/7	13*	1.40	11.0	1.0	0.17	D35	3/9	25	0.46	4.0	6.7	0.27
A4	7/12	62*	0.043	11.6+	10.2	0.31	D37	3/5	48	0.73	7.3	15.3	0.72
A6	2/4	17*	0.47	13.5+	17.0	0.57	D39	6/13	54	0.045	10.7	14.7	0.63
a80	3/6	33*	0.38	11.4+	5.3	0.22	D41	2/8	2	0.30	13.0	2.0	0.16
a81	2/4	14*	0.36	16.0	8.0	0.20	D43	3/6	33	0.69	9.3	9.3	0.24
a82	2/11	0.4*	0.105	11.5+	9.0	0.12	D45	4/9	47	1.24	11.5	5.0	0.51
a83	2/4	22*	0.30	18.0	7.0	0.023	D48	2/5	28	0.98	16.0	2.0	0.45
B9	2/10	6	0.99	7.0	4.0	0.35	e92	4/10	42	1.38	11.0	14.3	0.97
B10	2/4	19	0.83	27.0	2.0	0.024	E53	2/10	13	2.10	13.0	13.0	0.06
B12	3/7	49	0.91	12.7	6.0	0.32	E55	2/4	12	0.49	2.0	27.0	0.52
B14	2/6	0.46	0.045	0.0	5.0	0.033	E56	5/12	33	0.29	6.2	8.8	0.14
b85	2/8	5	0.62	5.0	9.0	0.34	E59	3/13	3.2	0.055	12.7	6.7	0.02
B17	2/6	31	2.61	4.0	13.0	0.095	E63	2/9	1.4	0.23	3.0	9.0	0.14
B19	2/6	11	1.06	21.0	3.0	0.010	E64	2/6	20	1.75	24.0	20.0	0.32
B20	2/5	9	0.63	3.0	2.0	0.12	E66	2/7	4.4	0.40	21.0	0.0	0.33
C21	6/15	66	0.25	7.7	10.4	0.66	E67	3/6	37	0.75	14.7	6.7	0.18
C25	3/13	2	0.026	6.7	8.7	0.15	E68	4/9	29	0.13	11.8	13.2	0.31
C26	2/7	4	0.59	15.0	16.0	0.24	E71	7/15	37	0.061	6.1	6.9	0.57
C27	2/8	1.5	0.31	5.0	3.0	0.01	E73	3/7	35	0.39	14.0	6.0	0.64
C28	2/10	0.02	0.003	6.0	0.0	0.15	F74	6/13	76*	0.48	10.3+	10.6	0.23
C31	4/12	15	0.22	14.0	5.5	0.62	F75	2/3	15*	0.25	10.0	4.0	0.46
C33	3/12	2	0.035	16.7	9.3	0.52	F76	4/6	60*	0.20	15.4+	9.7	0.26
c88	2/7	10	1.41	10.0	8.0	0.12	F77	2/2	11*	0.064	22.0	3.0	0.07
c87	3/12	24	1.03	10.0	14.7	0.36	F79	6/9	50*	0.047	13.0#	6.7	0.17

* $\Delta(\Omega \pm \omega)$

\+ Ω range = 60°

\# Ω range = 90°

Table 9

MEAN EXPECTATION (exp $\overline{\ln E} = E_1$)

Gp. or n	No. Boxes	E_1 σ
Tab. 3a	12	0.44 ± 0.09
15	20	0.31 ± 0.13
12	20	0.17 ± 0.06
10	20	0.29 ± 0.08
6	20	0.28 ± 0.12
All Random	92	0.28 ± 0.04
Öpik (Best)	50	0.33 ± 0.07

Table 10

MEAN CONCENTRATION IN BOXES

Data from Table 8 (Öpik, Min. E)

m	E	No.	$\overline{\Delta\Omega}$	σ	$\overline{\Delta i}$	σ	ΔTC	σ
All	All	50	11.2	± 0.8	8.9	± 0.8	0.29	± 0.03
>3	All	13	11.1	± 0.8	10.6	± 1.0	0.44	± 0.07
3	All	11	10.9	± 1.1	9.1	± 1.1	0.33	± 0.06
2	All	26	11.5	± 1.5	8.0	± 1.4	0.20	± 0.03
All	E≤0.31	21	10.5	± 1.2	7.8	± 0.8	0.26	± 0.05
All	E≤0.11	11	10.6	± 1.8	8.0	± 1.3	0.25	± 0.07
2	E≤0.31	9	9.8	± 2.4	5.3	± 1.2	0.13	± 0.05
2	E≤0.11	4	9.9	± 4.7	5.1	± 2.2	0.09	± 0.03

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ANOMALOUS TAILS OF COMETS.

I. A REVIEW OF PAST "EDGE-ON" APPEARANCES

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ABSTRACT

A cometary tail consisting of dust particles, whose diameters are mostly in the submillimeter and millimeter size ranges, is termed anomalous, when its apparent direction in the sky diverges considerably from the projected anti-solar direction. In space the anomalous tail is essentially a thin sheet of cometary debris confined to the orbit plane of the comet and located on the outside of the orbit and well behind the sun-comet direction. When the earth is crossing the orbit plane of the comet, the edge-on projection causes the anomalous tail to point exactly sunward and to adopt, typically, a spike-

like appearance. The conditions of projection for observing an anomalous tail at and around the time of crossing were formulated, and subsequently applied to list comets with favorable circumstances for displaying such anomalous tails. Since the production rate of dust and the particle size distribution are never known beforehand, it is always only the favorable observing situation and not the actual presence of the anomalous tail that can be predicted. An extensive search was undertaken for reports on observations of the expected anomalous tails, among both the nearly-parabolic comets (revolution periods over 200 years) and the short-period comets. It was found that of the nearly-parabolic comets that could have been observed to exhibit an anomalous tail, only every third to seventh comet was actually reported to have done so. The appearance of anomalous tails of the nearly-parabolic comets correlates strongly with aphelion distance, moderately with intrinsic brightness and spectrum, and possibly also with some other comet characteristics. These conclusions clearly have ramifications in the predictions of anomalous tails for the future nearly-parabolic comets. On the other hand, no certain observation of an anomalous tail of a short-period comet has ever been reported, although there were many opportunities. This finding is apparently inconsistent with the existence of meteor streams known to be associated

with a number of the short-period comets, as particles in anomalous tails and meteoroids producing radio meteors are mutually comparable in size. Although tentative explanations of the absence of correlation between meteor streams and anomalous tails are offered, the most certain way to resolve this controversy would be a systematic search for anomalous tails in the future returns of the short-period comets; such a project can be facilitated substantially by an advance comprehensive examination of favorable projection conditions.